CT Image Algebraic Reconstruction

What is Inverse Problem?

An **inverse problem** is a general framework that is used to convert observed measurements into information about a physical object or system that we are interested in. For example, if we have measurements of the Earth's gravity field, then we might ask the question: "given the data that we have available, what can we say about the density distribution of the Earth in that area?" The solution to this problem (i.e. the density distribution that best matches the data) is useful because it generally tells us something about a physical parameter that we cannot directly observe. Thus, inverse problems are some of the most important and well-studied mathematical problems in science and mathematics. Inverse problems arise in many branches of science and mathematics, including computer vision, natural language processing, machine learning,statistics, statistical inference, geophysics, medical imaging (such as computed axial tomography and EEG/ERP), remote sensing, ocean acoustic tomography, nondestructive testing, astronomy, physics and many other fields (Wikipedia).

What is Overdetermined System?

In mathematics, a system of linear equations is considered **overdetermined** if there are more equations than unknowns (Wikipedia).

What is Underdetermined System?

In mathematics, a system of linear equations or a system of polynomial equations is considered **underdetermined** if there are fewer equations than unknowns (in contrast to an overdetermined system, where there are more equations than unknowns) (Wikipedia).

HW Problems

In this homework, we try to reconstruct an image by solving a set of linear equations. Let's consider an original image as below,

4	7	5
8	1	3
6	2	9

I. You want to reconstruct this image from a set of projections. Since there are nine unknowns (or attenuation coefficients), you will need at least 9 equations. Make a set of nine projections of your choice and reconstruct the image by solving the set of nine equations (use pinv in Matlab, but why? Use the weight values of 1). Compute the sum of square error (SSE). Discuss about the error between the original and reconstructed image.

II. This time, make more than nine projections (say, ~12 or more and the weight values of 1). Reconstruct the image by solving this overdetermined problem. Compare the reconstruction results against the results of I. You should use pinv in Matlab (why?). See an example in two dimensions for the solutions of the overdetermined system at http://en.wikipedia.org/wiki/Overdetermined_system for your discussion.

III. Finally, try some different weight values of your choice (give some smart thinking) for the exact system in I and overdetermined system in II. Compare and discuss the results.