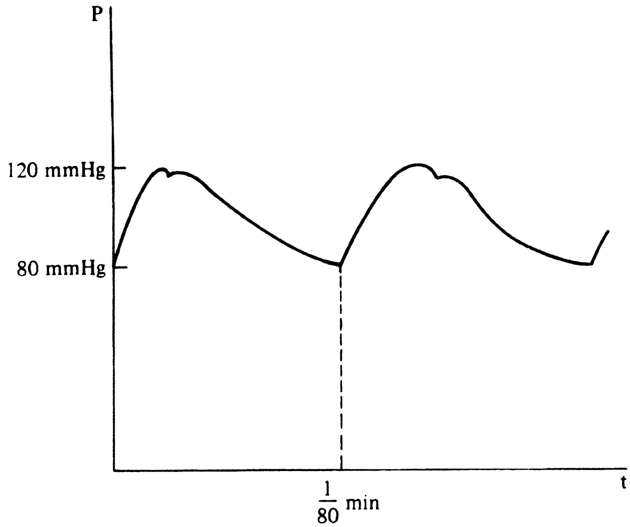

Figure 1.10. Systemic arterial blood pressure (P) as a function of time (t). The normal maximum (systolic) and minimum (diastolic) pressures are indicated on the pressure axis, and a typical period of the heartbeat is indicated on the time axis.

A self–consistent closed foramen solution is found in the Exercises at the end of the chapter. This describes the circulation after birth when $R_p < R_s$. One property of this solution is $Q_d < 0$, which means that the flow in the ductus has reversed compared to the fetal case. After birth the flow through the ductus is a *left-to-right shunt*, and it carries fully oxygenated blood from the aorta back into the pulmonary arterial tree. The high O_2 content of this blood stimulates the closure of the ductus, completing the transition to a single-loop circulation.

1.11 Dynamics of the Arterial Pulse

In the foregoing sections we have treated the circulation as though all of the pressures, flows, and volumes are constant in time. This is not correct. Actually, the heart ejects blood into the arteries in discrete bursts. During these contractions of the heart, the blood pressure rises rapidly, and it falls again between contractions as blood runs out of the arteries through the tissues. The result of this process is the *arterial pulse*, which can be felt wherever an artery is convenient to press (e.g., at the wrist) and which can be used to count the heart rate. The waveform of the arterial pulse is sketched in Figure 1.10.

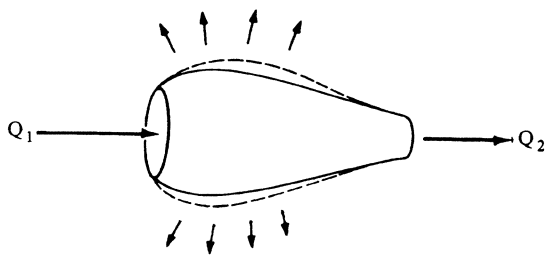

Figure 1.11. Volume conservation in pulsatile flow. The rate of change of volume of a vessel is equal to its inflow (Q_1) minus its outflow (Q_2).

When blood pressure is measured with an air cuff on the upper arm, the actual quantities determined by this measurement are the maximum (systolic) and minimum (diastolic) pressures achieved by the arterial pulse. A blood pressure of 120/80 means that the systolic arterial blood pressure is 120 mmHg and the diastolic arterial blood pressure is 80 mmHg. The difference between these values (in this case 40 mmHg) is called the *pulse pressure.*

The names *systolic* and *diastolic* refer to the phases of the cardiac cycle (see Section 1.4). Under normal conditions, the systolic pressure in the systemic arteries is essentially the same as the systolic left ventricular pressure, since the aortic valve is open during systole. The diastolic pressure in the ventricle is much lower than that in the arteries. This is possible because the aortic valve is closed during diastole.

In this section we shall describe the simplest model that can account for the qualitative form of the arterial pulse. This model will be used to show how the systolic and diastolic pressures depend on the parameters of the heart and circulation. It will also be used to justify the simpler steady flow models that were studied in the foregoing sections of this chapter.

We begin by considering a compliance vessel that is not in the steady state (see Figure 1.11). Thus, the inflow $Q_1(t)$ is not equal to the outflow $Q_2(t)$ at every instant. When they are not equal, the volume of the vessel changes. In fact, if $V(t)$ denotes the volume of the vessel at time t, we have a differential equation:

$$\dot{V} = Q_1 - Q_2. \tag{1.11.1}$$

(Here and below we write $\dot{V}$ for dV/dt.) This says that the rate of change of the vessel's volume is the difference between the flow in and the flow out. When V = constant, $Q_1 = Q_2$, which is the steady–state relation that we have used up to now.

This differential equation describes how the volume changes, but it can be converted into one relating the pressure in the vessel to the flows in and out by using the equation of a compliance vessel: Either

$$V(t) = CP(t) \quad \text{or} \quad V(t) = CP(t) + V_d$$

(see Section 1.3). In either case, we have $\dot{V} = C\dot{P}$, so

$$C\dot{P} = Q_1 - Q_2. \tag{1.11.2}$$

This is the equation that governs pressure changes in a compliance vessel in the case of unsteady flow.

Next, we use this equation to study the systemic arterial tree. Now, $P = P_{sa}$, the systemic arterial pressure; $C = C_{sa}$, the systemic arterial compliance, $Q_1 = Q_L$, the output of the left heart, and $Q_2 = Q_s$, the blood flow through the systemic tissues. For Q_s, we have the equation $Q_s = (P_{sa} - P_{sv})/R_s$, which we approximate by

$$Q_s = \frac{P_{sa}}{R_s} \tag{1.11.3}$$

since $P_{sv} \ll P_{sa}$. Thus, (1.11.2) becomes

$$C_{sa}\dot{P}_{sa} = Q_L(t) - \frac{P_{sa}}{R_s} \tag{1.11.4}$$

During diastole, when the aortic valve is closed, $Q_L = 0$. In that case, the solution of (1.11.4) is

$$P_{sa}(t) = P_{sa}(0)\exp\left(-\frac{t}{R_s C_{sa}}\right). \tag{1.11.5}$$

The constant $P_{sa}(0)$ remains to be determined.

To find it, we consider ventricular systole. We make the simplifying assumption that the entire stroke volume ΔV_0 is ejected from the heart instantaneously. Then we cannot use equation (1.11.4) for systole, at least not in an elementary way. We can, however, figure out what happens during systole by considering the change in arterial pressure produced in the arteries by a sudden change in volume of magnitude ΔV_0: From the compliance equation $V = CP + V_d$, it is clear that

$$\Delta V_0 = C_{sa}\Delta P_{sa}. \tag{1.11.6}$$

Now suppose that the heartbeat is a *periodic* phenomenon. That is, suppose that everything repeats exactly from one beat to the next. Let the duration of each heartbeat be T, so that the heart rate is $1/T$. Then the diastolic arterial pressure is $P_{sa}(T)$ and the systolic arterial pressure is $P_{sa}(0)$ (see Figure 1.12). Thus, the jump in pressure caused by cardiac ejection is given by the formula

$$\Delta P_{sa} = P_{sa}(0) - P_{sa}(T). \tag{1.11.7}$$

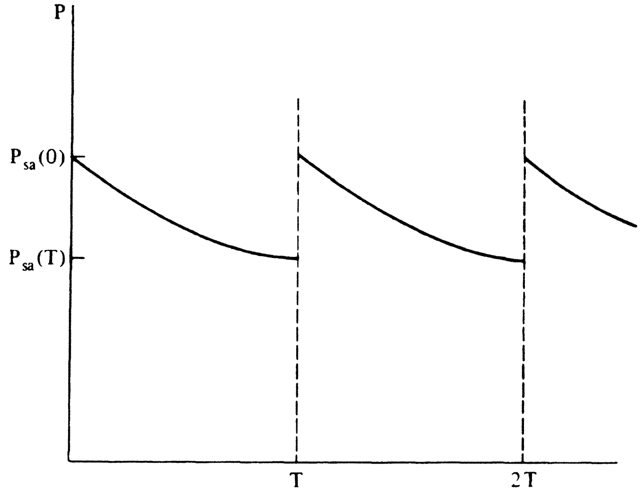

Figure 1.12. Idealized arterial pulse, under the assumption that ejection of the entire stroke volume from the left ventricle occurs instantaneously. T = period of the heartbeat, $P_{sa}(0)$ = systolic pressure, $P_{sa}(T)$ = diastolic pressure.

Now let $t = T$ in (1.11.5) and substitute (1.11.7) into (1.11.6) to obtain a pair of equations for $P_{sa}(0)$ and $P_{sa}(T)$. The result is

$$P_{sa}(T) = \Theta P_{sa}(0), \tag{1.11.8}$$

$$\frac{\Delta V_0}{C_{sa}} = P_{sa}(0) - P_{sa}(T), \tag{1.11.9}$$

where

$$\Theta = \exp\left(-\frac{T}{(R_s C_{sa})}\right). \tag{1.11.10}$$

Note that $0 < \Theta < 1$. Solving these equations gives

$$P_{sa}(0) = \frac{\Delta V_0}{C_{sa}(1-\Theta)}, \tag{1.11.11}$$

$$P_{sa}(T) = \frac{\Delta V_0 \Theta}{C_{sa}(1-\Theta)}, \tag{1.11.12}$$

which are formulae for the systolic and diastolic pressures in terms of the stroke volume, the arterial compliance, the systemic resistance, and the heart rate. Subtracting these two equations, we recover (1.11.9), which is the formula for the pulse pressure.

What about the mean arterial pressure? A useful definition of the *mean* of a periodic function, say $f(t)$, is

$$\langle f \rangle = \frac{1}{T}\int_0^T f(t)\,dt,$$

where the period of f is T and where integration is over any period, e.g., over the interval $(0,T)$. With this notation, we define the mean arterial pressure to be

$$\langle P_{\mathrm{sa}}\rangle = \frac{1}{T}\int_0^T P_{\mathrm{sa}}(t)\,dt. \tag{1.11.13}$$

(This is approximately the average of N samples of the function $P_{\mathrm{sa}}(t)$ taken at equally spaced times that span the interval $[0,T]$. The approximation becomes exact as $N \to \infty$.) We leave it as an exercise to check that

$$\langle P_{\mathrm{sa}}\rangle = \frac{R_{\mathrm{s}}\Delta V_0}{T}. \tag{1.11.14}$$

Since $\Delta V_0/T$ is the cardiac output, this equation can be interpreted as $\langle P_{\mathrm{sa}}\rangle = QR_{\mathrm{s}}$, which is the equation that holds in the steady state case (if we neglect P_{sv} as we are doing here). This explains how a steady–state model still has significance for a pulsatile circulation: The quantities appearing in the steady–state model are the time averages of the corresponding pulsatile quantities.[1]

We have determined the form of the arterial pulse in the periodic case, where everything repeats exactly from one beat to the next. This is not quite correct even in the normal circulation, where the heart rate and stroke volumes change slightly in response to the phases of breathing. The assumption of periodicity is even less appropriate for individuals with abnormal rhythms of the heart where successive heartbeats may be considerably different from each other, both in their durations and in their stroke volumes. As an extreme example of an aperiodic arterial pulse, consider what happens if the heart has just been started following a period of cardiac arrest. The arterial pressure is initially very low, and it has to build up toward its equilibrium values over the first few beats. The rest of this section studies these transient situations.

If the stroke volume and timing vary from beat to beat, we need notation to tell us what happens on each beat. Let $j = 1, 2, \ldots$ be an index counting beats of the heart. Let t_j be the time that the jth beat occurs, and let ΔV_j be the corresponding stroke volume. Since the pressure $P_{\mathrm{sa}}(t)$ jumps at the times t_j (see Figure 1.13), we need notation to distinguish the arterial pressures just before and just after cardiac ejection. Let

$P_{\mathrm{sa}}(t_j^-)$ = arterial pressure just before ejection (diastolic);

[1] This statement is only approximate. In a pulsatile version of the whole circulation model considered above, the pressures that determine the cardiac outputs would be the end-diastolic venous pressures, not the mean pressures, since it is the end-diastolic pressures that determine the volumes of the ventricular chambers just prior to ejection. More generally, any nonlinearity that might be introduced to make the model more realistic would further degrade the correspondence between the steady–flow results and the mean values of the pulsatile results.

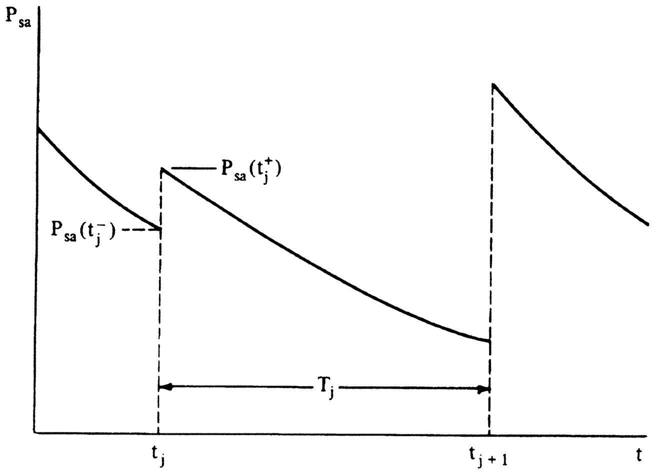

Figure 1.13. Idealized arterial pulse in the presence of an irregular heart rhythm. t_j = time of the jth ventricular systole (idealized here as instantaneous), $P_{\text{sa}}(t_j^-)$ = diastolic pressure immediately before the jth ventricular systole, $P_{\text{sa}}(t_j^+)$ = systolic pressure immediately after the jth ventricular systole.

$P_{\text{sa}}(t_j^+)$ = arterial pressure just after ejection (systolic).

Between beats of the heart, we have, as before, the differential equation

$$C_{\text{sa}}\dot{P}_{\text{sa}} = -P_{\text{sa}}/R_{\text{s}},$$

but now it is more convenient to write the solutions in the form

$$P_{\text{sa}}(t) = P_{\text{sa}}(t_j^+)\exp(-(t-t_j)/(R_{\text{s}}C_{\text{sa}})) \qquad (1.11.15)$$

for $t_j < t < t_{j+1}$. Setting $t = t_{j+1}^-$ gives

$$P_{\text{sa}}(t_{j+1}^-) = P_{\text{sa}}(t_j^+)\Theta_j, \qquad (1.11.16)$$

where

$$\Theta_j = \exp(-T_j/(R_{\text{s}}C_{\text{sa}})) \qquad (1.11.17)$$

and

$$T_j = t_{j+1} - t_j. \qquad (1.11.18)$$

Equation (1.11.16) gives the diastolic pressure just before beat $j+1$ in terms of the systolic pressure just after beat j.

The equation for the jump in arterial pressure on beat j now takes the form

$$P_{\text{sa}}(t_j^+) = P_{\text{sa}}(t_j^-) + \Delta V_j/C_{\text{sa}}. \qquad (1.11.19)$$

Now suppose that we are given any sequence of times t_j and stroke volumes ΔV_j together with the constant parameters C_{sa} and R_s. If we are told the diastolic pressure just before the first beat, we can use (1.11.19) to find the systolic pressure just after that beat. Then we can use (1.11.16) to find the diastolic pressure just before the next beat. Repeating this process we can predict the entire sequence of diastolic and systolic pressures, however irregular it might be.

The equations that we have just developed for the aperiodic situation should include the periodic arterial pulse as a special case. Suppose the heartbeat is regular, so that $t_{j+1} - t_j = T$ and $\Delta V_j = \Delta V_0$ for all j. Then Θ_j reduces to Θ, and equations (1.11.16) and (1.11.19) become

$$P_{sa}(t_{j+1}^-) = P_{sa}(t_j^+)\Theta, \tag{1.11.20}$$

$$P_{sa}(t_j^+) = P_{sa}(t_j^-) + \Delta V_0/C_{sa}. \tag{1.11.21}$$

Now we can look for a solution of these equations in which $P_{sa}(t_j^+)$ and $P_{sa}(t_j^-)$ are independent of j. We express this by means of the notation

$$P_{sa}(t_j^+) = P_{sa}^+ \text{ (systolic pressure)}, \tag{1.11.22}$$

$$P_{sa}(t_j^-) = P_{sa}^- \text{ (diastolic pressure)}. \tag{1.11.23}$$

Thus, P_{sa}^+ and P_{sa}^- satisfy

$$P_{sa}^- = P_{sa}^+\Theta \tag{1.11.24}$$

$$P_{sa}^+ = P_{sa}^- + \Delta V_0/C_{sa}. \tag{1.11.25}$$

These are the same equations as (1.11.8) and (1.11.9) for the periodic systolic and diastolic pressures. This confirms that our theory of irregular arterial pulses contains the periodic pulse as a special case. (There are other solutions of (1.11.20) and (1.11.21) that do not correspond to a periodic pressure pulse even though the heartbeat and stroke volume are regular. Can you show that these solutions approach the periodic pulse as time increases?)

1.12 Computer Simulation of Pulsatile Blood Flow

In this section we show how computers can be used to solve the equations of time dependent pressure, flow, and volume in the circulatory system. We start with a simple case, the systemic arterial pulse, and work up to a model of the circulation as a whole. Already in the case of the arterial pulse, we shall see that the use of computers gives us the freedom to make our mathematical models more complicated and more realistic than before.

The equations of the systemic arterial pulse that we shall use in this section are as follows:

$$\dot{V}_{sa} = Q_{Ao} - Q_s, \tag{1.12.1}$$

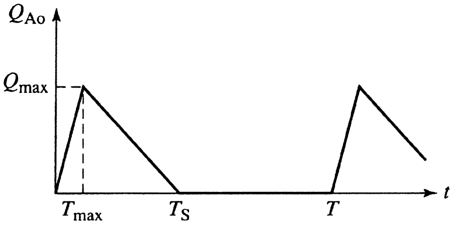

Figure 1.14. Outflow from the left ventricle trough the aortic valve into the systemic arterial tree is here denoted $Q_{Ao}(t)$. It is a periodic function whose triangular waveform is determined by the parameters Q_{max} = peak flow, T_{max} = time of peak flow, T_S = duration of systole, and T = duration of heartbeat. Note that the stroke volume is given by the integral of $Q_{Ao}(t)$ over one heartbeat, which is equal to $Q_{max}T_S/2$, and that the cardiac output is the stroke volume times the heart rate, which is therefore equal to $Q_{max}T_S/(2T)$.

$$V_{sa} = (V_{sa})_d + C_{sa}P_{sa}, \tag{1.12.2}$$

$$Q_s = P_{sa}/R_s. \tag{1.12.3}$$

In these three equations, the three unknowns (all of which are functions of time) are V_{sa}, the systemic arterial volume; P_{sa}, the systemic arterial pressure; and Q_s, the flow through the systemic resistance. Note that Q_s can also be described as the outflow from the systemic arteries. The systemic arterial pulse is driven by the inflow to the systemic arteries, which is the flow through the aortic valve, denoted here by Q_{Ao}. This is a function of time, which for purposes of modeling just the systemic arterial pulse we shall treat as a *given* function of time. Its specific form will be specified later. The parameters appearing in these equations are $(V_{sa})_d$, the volume of the systemic arteries when $P_{sa} = 0$; C_{sa}, the compliance of the systemic arteries; and R_s, the systemic resistance. These three parameters are all (given) constants. (But $(V_{sa})_d$ drops out of the equations when we express everything in terms of the pressure, below.)

Equation (1.12.1) expresses conservation of volume for the systemic arteries: The rate of change of volume is the inflow minus the outflow. Equation (1.12.2) is the compliance relationship between systemic arterial pressure and volume. Equation (1.12.3) describes the flow through the systemic resistance in terms of the systemic arterial pressure. In writing (1.12.3), we have made the approximation of neglecting the systemic venous pressure in comparison to the systemic arterial pressure.

By substituting equations (1.12.2) and (1.12.3) into equation (1.12.1), we reduce the above system to a single equation for the systemic arterial pressure:

$$C_{sa}\dot{P}_{sa} = Q_{Ao} - P_{sa}/R_s. \tag{1.12.4}$$

Except for slight differences in notation, the theory that we have outlined here is the same as in Section 1.11, but with this important difference: Here we allow Q_{Ao} to be any (given) function of time. The specific function that we shall use is shown in Figure 1.14 and may be described mathematically as follows. First, Q_{Ao} is a periodic function of time:

$$Q_{\text{Ao}}(t+T) = Q_{\text{Ao}}(t), \tag{1.12.5}$$

where T is the period of the heartbeat ($1/T$ is the heart rate). Because Q_{Ao} is periodic, we can define it completely by specifying it in any one period, say $(0, T)$. This is done as follows:

$$Q_{\text{Ao}}(t) = \begin{cases} Q_{\max} t/T_{\max}, & 0 \le t \le T_{\max} \\ Q_{\max}(T_{\text{S}} - t)/(T_{\text{S}} - T_{\max}), & T_{\max} \le t \le T_{\text{S}} \\ 0, & T_{\text{S}} \le t \le T \end{cases} \tag{1.12.6}$$

where $Q_{\max}$ is the maximum flow, $T_{\max}$ is the time at which maximum flow occurs (relative to the beginning of systole), and T_{S} is the duration of systole. Note that the stroke volume (integral of Q_{Ao} over one cycle) is given by $Q_{\max}T_{\text{S}}/2$ and that the cardiac output is therefore given by $Q_{\max}T_{\text{S}}/(2T)$. Reasonable choices of the parameters that determine Q_{Ao} are $T = 0.0125$ minutes, $T_{\text{S}} = 0.0050$ minutes, $T_{\max} = 0.0020$ minutes, $Q_{\max} = 28$ liters/minute. Note that this gives a stroke volume of 0.070 liters and a cardiac output of 5.6 liters/minute.

The next step is to introduce a numerical method for the (approximate) solution of equation (1.12.4). Such a method can be found by replacing the time derivative in that equation by a difference quotient. There are several ways to do this, but we choose a "backward" method, for reasons that will be discussed later (see the discussion following equation 1.12.33). When we do this, equation (1.12.4) becomes

$$C_{\text{sa}}(P_{\text{sa}}(t) - P_{\text{sa}}(t - \Delta t))/\Delta t = Q_{\text{Ao}}(t) - P_{\text{sa}}(t)/R_{\text{s}}. \tag{1.12.7}$$

The way that we shall use this equation is to solve it for $P_{\text{sa}}(t)$ in terms of $P_{\text{sa}}(t - \Delta t)$ and other known quantities, including $Q_{\text{Ao}}(t)$. The solution is:

$$P_{\text{sa}}(t) = \frac{P_{\text{sa}}(t - \Delta t) + \Delta t\, Q_{\text{Ao}}(t)/C_{\text{sa}}}{1 + \Delta t/(R_{\text{s}}C_{\text{sa}})}. \tag{1.12.8}$$

Now if we have a starting value $P_{\text{sa}}(0)$, we can use (1.12.8) repeatedly to find $P_{\text{sa}}(\Delta t)$, $P_{\text{sa}}(2\Delta t)$, $P_{\text{sa}}(3\Delta t)$, and so on, until we have spanned whatever interval of time may be of interest.

Two questions may be bothering the reader at this point: How should I choose the starting value $P_{\text{sa}}(0)$, and how should I choose the time step Δt?

The answer to the first question is that you can choose any value at all for $P_{\text{sa}}(0)$. No matter what pressure you start at, the arterial pulse will eventually settle down into the same periodic waveform. When you get your computer program working, try different starting values and compare the results. The arterial pulse will gradually "forget" what the starting value was. So you don't have to worry about this, except that you have to specify some starting value, and you have to run your program long enough for the pulse waveform to settle down. Now you might say in response to this, "Why don't I just start at the diastolic pressure, since I know what that is (80 mmHg)?" The trouble with this is that you know the diastolic pressure only under normal conditions. A computer program such as the one we are creating here wouldn't be of much use if it could only handle one case. What you want to do is to change parameters and see what happens. As soon as you change parameters, you don't know the diastolic pressure anymore, so you will just have to let the program run until the pressure waveform settles into its periodic pattern.

The answer to the second question is that Δt has to be "small enough." Recall that we made an approximation in replacing $\dot{P}_{\text{sa}}$ by $(P_{\text{sa}}(t) - P_{\text{sa}}(t-\Delta t))/\Delta t$. This approximation gets better as Δt gets smaller. To tell whether Δt is small enough, you should rerun your program with Δt replaced by $\Delta t/2$. (Note that with Δt half as big you will have to take twice as many steps to span the same amount of time.) If the results are essentially the same (to within whatever tolerance you think is reasonable), then Δt is indeed small enough. If not, then you have to keep making Δt smaller. To get into the right ballpark in the first place, you should choose Δt small compared to any of the times that define the problem. In our case, a reasonable choice might be $\Delta t = 0.01T$. With this choice, it takes 100 steps to span a cardiac cycle.

The numerical method that we have just outlined is implemented by the following Matlab program. It involves one script and two functions. We shall describe the functions first, so it will be clear what the script is doing when it calls these functions. The functions are `QAo_now(t)`, which computes the flow through the aortic valve at time t according to equations (1.12.5–1.12.6), and `Psa_new(Psa_old,QAo)`, which computes the "new" pressure $P_{\text{sa}}(t)$ in terms of the "old" pressure $P_{\text{sa}}(t-\Delta t)$ and the given flow $Q_{\text{Ao}}(t)$ through the aortic valve. These functions read as follows:

```
function Q=QAo_now(t)
%filename: QAo_now.m
global T TS TMAX QMAX;
tc=rem(t,T); % tc=time elapsed since
%the beginning of the current cycle
%rem(t,T) is the remainder when t is divided by T
if(tc<TS)
  %SYSTOLE:
  if(tc<TMAX)
    %BEFORE TIME OF MAXIMUM FLOW:
    Q=QMAX*tc/TMAX;
  else
```

```
    %AFTER TIME OF PEAK FLOW:
    Q=QMAX*(TS-tc)/(TS-TMAX);
  end
else
  %DIASTOLE:
  Q=0;
end
```

```
function Psa=Psa_new(Psa_old,QAo)
%filename:   Psa_new.m
global Rs Csa dt;
Psa=(Psa_old+dt*QAo/Csa)/(1+dt/(Rs*Csa));
```

With these two functions in hand, we can write a script that we shall call sa for "systemic arteries." It loops over time and computes $P_{sa}(t)$. It will call another script in_sa that does initialization. These scripts read as follows:

```
%filename: sa.m
clear all % clear all variables
clf       % and figures
global T TS TMAX QMAX;
global Rs Csa dt;
in_sa %initialization
for klok=1:klokmax
  t=klok*dt;
  QAo=QAo_now(t);
  Psa=Psa_new(Psa,QAo); %new Psa overwrites old
  %Store values in arrays for future plotting:
  t_plot(klok)=t;
  QAo_plot(klok)=QAo;
  Psa_plot(klok)=Psa;
end
%Now plot results in one figure
%with QAo(t) in upper frame
% and Psa(t) in lower frame
subplot(2,1,1), plot(t_plot,QAo_plot)
subplot(2,1,2), plot(t_plot,Psa_plot)
```

```
%filename: in_sa.m  (initialization for the script sa)
T =0.0125     %Duration of heartbeat (minutes)
TS=0.0050     %Duration of systole   (minutes)
TMAX=0.0020   %Time at which flow is max (minutes)
QMAX=28.0     %Max flow through aortic valve (liters/minute)
Rs=17.86      %Systemic resistance (mmHg/(liter/minute))
Csa=0.00175   %Systemic arterial compliance (liters/(mmHg))
%This value of Csa is approximate and will need adjustment
```

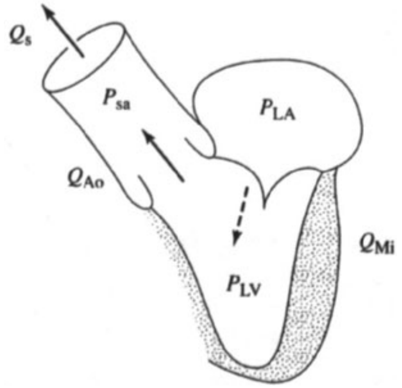

Figure 1.15. Model of the left heart and systemic arteries. Pressures and flows are P_{LA} = left atrial pressure, P_{LV} = left ventricular pressure, P_{sa} = systemic arterical pressure, Q_{Mi} = flow through the mitral valve (indicated here by a dotted arrow because the mitral valve is closed at the moment shown), Q_{Ao} = flow through the aortic valve, and Q_s = outflow from the systemic arterial tree, i.e., aggregate flow through all of the tissues of the body.

```
%to make the blood pressure be 120/80.
dt=0.01*T    %Time step duration (minutes)
%This choice implies 100 timesteps per cardiac cycle.
klokmax=15*T/dt %Total number of timesteps
%This choice implies simulation of 15 cardiac cycles.
Psa=0           %Initial value of Psa (mmHg)
%Any initial value is OK here; try some others.
%Initialize arrays to store data for plotting:
  t_plot=zeros(1,klokmax);
QAo_plot=zeros(1,klokmax);
Psa_plot=zeros(1,klokmax);
```

The program for the systemic arterial pulse is now complete. It is the collection of four files `sa.m`, `in_sa.m`, `QAo_now.m`, and `Psa_new.m`. To invoke it from within a Matlab session launched from the same directory where these files reside, just type "sa" at the Matlab prompt ">>". Physiological applications of this computer program are outlined in Section (1.13).

The next step in building up towards a computer model of the whole circulation is to attach the left heart to the systemic arteries. This model is shown in Figure 1.15. The model of the left heart that we shall use was already outlined in Section 1.4; it treats the left ventricle as a compliance vessel the compliance of which is not constant but instead is some given function of time. This given function $C_{LV}(t)$ will replace $Q_{Ao}(t)$ as the periodic time function that drives the cardiac cycle, and $Q_{Ao}(t)$ will become one of the unknowns of the model.