The Heart & Circulation (II)

(In Modeling and Simulation in Medicine and the Life Sciences, 1.5 to 1.9)

1.5 Mathematical Model of the <u>Uncontrolled</u> Circulation

$$Q_R = K_R P_{SV}$$
$$Q_L = K_L P_{PV}$$
$$V_i = C_i P_i, i = sa, sv, pa, pv$$
$$Q_s = \frac{1}{R_s}(P_{sa} - P_{sv})$$
$$Q_p = \frac{1}{R_p}(P_{spa} - P_{pv})$$

- 8 Equations and 12 unknowns, $Q_R, Q_L, Q_s, Q_p, P_{sa}, P_{sv}, P_{pa}, P_{pv}, V_{sa}, V_{sv}, V_{pa}, V_{pv}$
- Cannot solve for the unknowns

Assume

The total blood volume, $V_0 = V_{sa} + V_{sv} + V_{pa} + V_{pv}$

$$Q = Q_R = Q_L = Q_s = Q_p$$

- 9 equations and 9 unknowns, $Q, P_{sa}, P_{sv}, P_{pa}, P_{pv}, V_{sa}, V_{sv}, V_{pa}, V_{pv}$

$$V_i = T_i Q, i = sv, pv, sa, pa$$

$T_{sv}, T_{pv}, T_{sa}, T_{pa}$ given as (1.5.18-1.5.21)

$$P_i = V_i / C_i, i = sv, pv, sa, pa$$

$$V_0 = (T_{sv} + T_{pv} + T_{sa} + T_{pa})Q$$

R, C, K, V_0 given Table 1.2

The model is complete now.

1.6 Modeling Two Sides of the Heart and the Two Circulations

Partition the heart into the right and left sides (assume the pulmonary and systemic volumes (Vp, Vs) were separately given)

$$V_s = V_{sa} + V_{sv}$$

$$V_p = V_{pa} + V_{pv}$$

$$V_i = C_i P_i, i = sa, sv, pa, pv$$

$$Q_s = \frac{1}{R_s}(P_{sa} - P_{sv})$$

$$Q_p = \frac{1}{R_p}(P_{pa} - P_{pv})$$

- 8 equations and 8 unknowns, $P_{sa}, P_{sv}, P_{pa}, P_{pv}, V_{sa}, V_{sv}, V_{pa}, V_{pv}$ given R_s, R_p, C_i in Table 1.2
- Note the flow (or cardiac output dependence on the pressure difference). It's uncontrolled yet.

1.7 The Need for External Circulatory Control Mechanism

When the arterioles dilate, the systemic resistance R_s decreases. Then the cardiac output increases, while the systemic arterial pressure is maintained.

The increase in cardiac output primarily comes from an increase in heart rate while stroke volume remains fairly constant.

Given

$$Q = \frac{V_0}{T_{sv} + T_{pv} + T_{sa} + T_{pa}}$$

And

$$P_{sa} = \frac{V_0}{C_{sa}} \frac{T_{sa}}{T_{sv} + T_{pv} + T_{sa} + T_{pa}}$$

If we reduce R_s by 50%, Q only increases 10% and P_{sa} changes 40%.

Let's further modify to correct this problem

1.8 Neural Control: The Basoreceptor Loop

In the body, P_{sa} is controlled by a feedback mechanism called the baroreceptor loop

See Figure 1.7

- The baroreceptors are located in the carotid arteries and in the arch of the aorta.
- The baroreceptors transmit nerve impulses to the brain stem at a rate that increases with increasing arterial pressure
- Parasympathetic nervous system slows the heart rate (F)
- Sympathetic nervous system increases heart rate, venous pressure, and systemic resistance

Let's assume there is a target pressure P^* for the systemic arterial pressure such that

$$P_{sa} = P^*$$

$$Q_R = FC_R P_{sv}$$

$$Q_L = FC_L P_{pv}$$

$$V_i = C_i P_i, i = sa, sv, pa, pv$$

$$Q_s = \frac{1}{R_s}(P_{sa} - P_{sv})$$

$$Q_p = \frac{1}{R_p}(P_{pa} - P_{pv})$$

$$V_0 = V_{sa} + V_{sv} + V_{pa} + V_{pv}$$

$$Q = Q_R = Q_L = Q_s = Q_p$$

- Same unknowns, except P_{sa} is replace by P^* that should be known as a target.
- F is a new parameter

Further assumption

- Since P_{sv} small compared to P_{sa}, ignore P_{sv} (2% error)
- $QR_s = P^*$

- Ignore the pulmonary volumes, V_{pa} and V_{pv} (10% error)
- $V_0 = V_{sa} + V_{sv}$
- Then, $C_{sa}P^* + C_{sv}P_{sv} = V_0$

Now

$$Q = \frac{P^*}{R_s}$$

$$P_{sv} = \frac{V_0 - C_{sa}P^*}{C_{sv}}$$

$$F = \frac{P^* C_{sv}}{R_s C_R (V_0 - C_{sa}P^*)}$$

Now this is the controlled circulation: $P_{sa} = P^*$ that is constant. F is controlled by R_s.

1.9 Autoregulation

What is autoregulation

- There is a range of pressures in which the flow is relatively insensitive to the pressure difference
- At constant pressure difference, the flow through many tissues depends on the rate of O_2 consumption of the tissue

Let's model the second part.

Let the concentration of O_2 in blood be $[O_2]$, M (metabolic rate) the rate of O_2 consumption of a tissue, Q the blood flow.

Then the rate of O_2 delivered to tissue in the arterial blood is $Q[O_2]_a$. In the steady state the difference in arterial and venous blood is then

$Q[O_2]_a - Q[O_2]_v = M$ (Fick's principle)

Rewrite $[O_2]_v = [O_2]_a - M/Q$ and assume $R = R_0[O_2]_v$ and $R = P/Q = (P_{sa} - P_{sv})/Q$.

The resistance is controlled by O_2 concentration.

Then $Q = \frac{M}{[O_2]_a} + \frac{P}{R_0[O_2]_a} = Q^* + \frac{P}{R_0[O_2]_a}$, given the minimum flow Q^*

By having $R = R_0[O_2]_v$ instead of R only as a constant, makes the blood supply responsive to the needs of the blood.