Homework # 3 Solutions

Problem 2

(a)
To solve this problem, Ho-kashyap pseudoinverse procedure is implemented.
The implemented algorithm is:

$\mathbf{b}(1) > 0$
$\mathbf{w}(1)=\mathbf{Y}^{+}\mathbf{b}(1)$
$\mathbf{b}(k+1)=\mathbf{b}(k)+\alpha(\mathbf{e}(k)+|\mathbf{e}(k)|)$
$\mathbf{w}(k+1)=\mathbf{w}(k)+\alpha\ \mathbf{Y}^{+}|\mathbf{e}(k)|$

where $\mathbf{e}(k)=\mathbf{Y}\mathbf{w}(k)-\mathbf{b}(k)$ and $\mathbf{Y}^{+}$ is the pseudoinverse of $\mathbf{Y}$.

The following is the Matlab implementation.

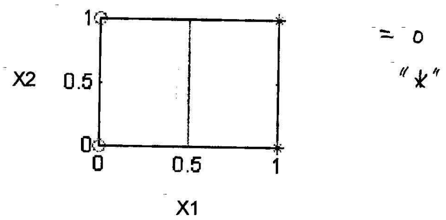
```
% hw3_2a.m
% Homework #3 Problem #3 2 (a)

% Augmented and reflected prototypes
Y=[0 0 1
   0 1 1
   -1 0 -1
   -1 -1 -1];

% b(1)=initial target vector
b=[1 1 1 1]';

% w(1)
w=pinv(Y)*b;

% bmin for stopping criteria
bmin=0.001;

for i=1:500
   error=Y*w-b;
   b=b+(abs(error)+error);
   w=w+pinv(Y)*abs(error);
   if min(abs(error))<bmin
      w
      b
      Yw=Y*w
      break
   end
end
```

The algorithm converges after one iteration with the weight vector
w =
-2.0000
0.0000
1.0000

To check the weight vector
Yw =[1.0000 1.0000 1.0000 1.0000]'=b=[1 1 1 1]'>0

(b)

Based on the weight vector,
The discriminant function g=-2*x1 + 0*x2 + 1.
Therefore the boundary becomes x1=1/2



(c)

The same algorithm is applied only with the different prototypes **Y**
The Matlab code is;

```
Y=[0 0 1
   0 1 1
   0 -1 -1
   -1 0 -1];
b=[1 1 1 1]';
w=pinv(Y)*b;
bmin=0.001;

   i=1:500
  i
  error=Y*w-b;
  b=b+(abs(error)+error);
  w=w+pinv(Y)*abs(error);
  if min(abs(error))<bmin
     w
     b
     Yw=Y*w
     break
```

The algorithm stops after 1 iteration with
w =

 -2.0000
 -1.0000
 1.0000
but
Yw =[1.0000 0.0000 0.0000 1.0000]' $\neq$ b $\not>$ 0
Therefore it tells there is no solution
(d) The resulting weight vector does not separate the prototypes. The problem is not separable.

2. (a) The optimal decision boundary is x1+ x2=3/2

(b) The optimal margin is sqrt(2)/4.