Pattern Classification Homework 2 Solutions

1.

(a). Yes, they are separable by a higher order polynomial discriminant functions.

The 2nd order polynomial function would separate them.

(b) Let $\phi = w_1 x_1^2 + w_2 x_2^2 + w_3 x_1 x_2 + w_4 x_1 + w_5 x_2 + w_6$

Then $g(x) = w^T x = [w_1\ w_2\ w_3\ w_4\ w_5\ w_6][\ x_1^2\ \ x_2^2\ \ x_1 x_2\ \ x_1\ x_{2\ 1}]^T$

Convert and reflect prototypes according to this.

Then (1 1 1 1 1 1)

(1 1 -1 1 -1 1)

(16 25 20 4 5 1)

(-4 -4 -4 -2 -2 -1)

(0 -4 0 0 -2 -1)

(-4 -9 -6 -2 -3 -1)

(c) Implement the perceptron algorithm.

(c) Find the weight vector

The Perceptron algorithm is coded with the step size alpha=1
Source Code executable in MATLAB is given below

```
START ----------------------------------------
%hw2 2c.m

%reflected prototypes
%Each row is one prototype
yy=[1 1 1 1 1 1
   1 1 -1 1 -1 1
   16 25 20 4 5 1
   -4 -4 -4 -2 -2 -1
   0 -4 0 0 -2 -1
   -4 -9 -6 -2 -3 -1];

% Initial weight vector
w=[-1 -1 -1 1 1 1];

% Maximum iteration for epoch is set to be 2000
for epoch=1:2000
 for i=1:6
      y=yy(i,:)';
      g=w*y;
      if g<=0
         w=w+y';  % alpha=1;
      end
      if g>0
         w=w;
      end
      k(i)=g;
 end

 if min(k)>0
    min(k)
    w
    break
 end
 end % for epoch

END ---------------------------------
```

The algorithm converges at 245 iterations or epochs.
The weight vector w is obtained as
W=[8 -6 24 -22 -66 63]

(d) Plot the discriminant function
To plot the discriminant function, ϕ must be solved for 0.
To solve this the symbolic toolbox in MATLAB is used.
Let x1=x, x2=y
In Matlab do this

```
>> solve('8*x^2-6*y^2+24*x*y-22*x-66*y+63');
```

this returns answer for y

The following x1 and x2 are the answers for y

```
x1=-3/2*y+11/8+1/8*(192*y.^2+264*y-383).^(1/2)
x2=-3/2*y+11/8-1/8*(192*y.^2+264*y-383).^(1/2)
```

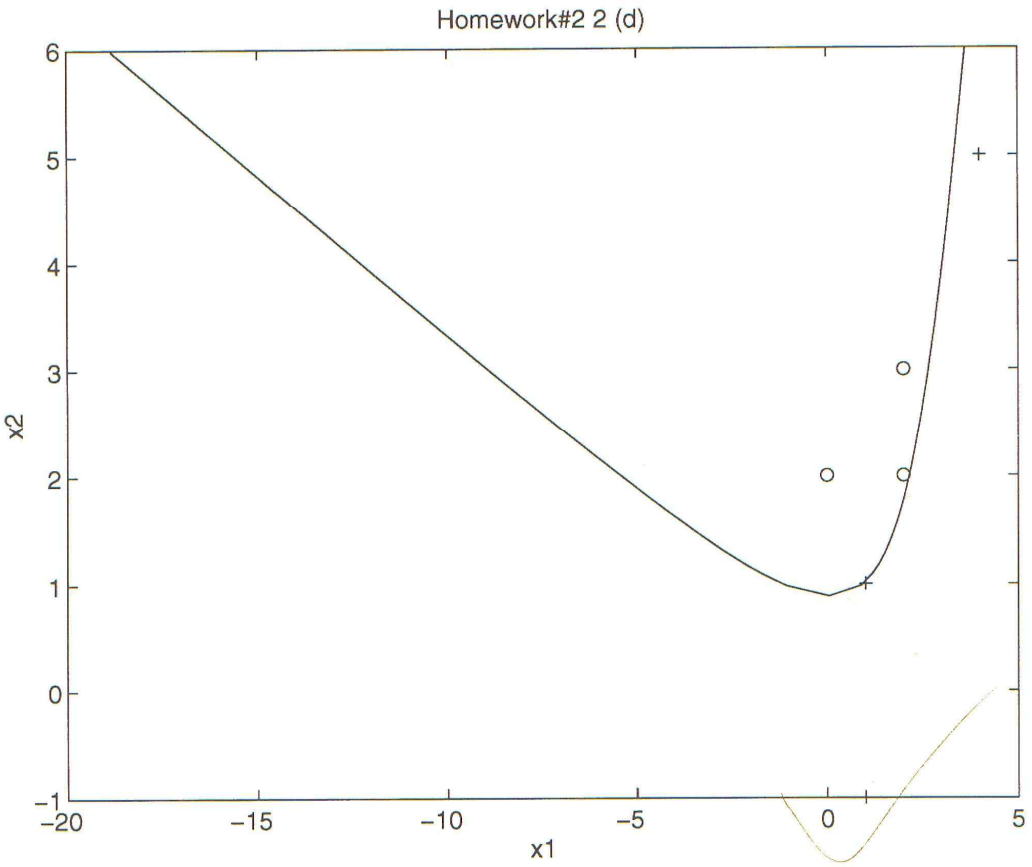
to plot the discriminant function, the following code is written

START----------

```
% to solve for the roots for the quadratic terms in x1 and x2
tmp1=roots([192 264 -383])
max_roots=max(tmp1);

% sample y values
y=max_roots:0.1:6

% compute x1 and x2 for each y
x1=-3/2*y+11/8+1/8*(192*y.^2+264*y-383).^(1/2)
x2=-3/2*y+11/8-1/8*(192*y.^2+264*y-383).^(1/2)

plot(x1,y,x2,y)
hold

% Class S1 and S2
s1xy=[ 1  1
       1 -1
       4  5];
s2xy=[2 2
      0 2
      2 3];


plot(s1xy(:,1),s1xy(:,2),'+',s2xy(:,1),s2xy(:,2),'o');
plot(s1xy(:,1),s1xy(:,2),'+',s2xy(:,1),s2xy(:,2),'o');
xlabel('x1')
ylabel('x2')
title('Homework#2 2 (d)')
```

END ----------

The plot is given at the next page.


Homework#2 2 (d)
6
5
4
3
2
1
0
−1
x2
−20
−15
−10
−5
0
5
x1