Pattern Classification Homework 1 Solutions

1. The discriminant functions are

$g_1(\underline{x})=-x_1+x_2$

$g_2(\underline{x})= x_1+x_2-1$

$g_3(\underline{x})= -x_2$

Then assign $\underline{x}$ to S_k iff $g_k(\underline{x}) > g_j(\underline{x})$ for all $j\neq k$

(a) Draw the decision boundaries. Label Classified regions

$g_1(\underline{x})=-x_1+x_2=[-1\ 1\ 0][x_1\ x_2\ 1]^T = \underline{w}^T\underline{x}^{(a)}$, $w_1^T=[-1\ 1\ 0]$

$g_2(\underline{x})= [1\ 1\ -1][x_1\ x_2\ 1]^T = \underline{w}^T\underline{x}^{(a)}$, $w_2^T=[1\ 1\ -1]$

$g_3(\underline{x})= [0\ -1\ 0][x_1\ x_2\ 1]^T = \underline{w}^T\underline{x}^{(a)}$, $w_3^T=[0\ -1\ 0]$

The decision boundary for S_1 over S_2

$(w_1^T- w_2^T)\underline{x}^{(a)}=0$, $[-2\ 0\ 1][x_1\ x_2\ 1]^T=0$, $-2x_1+1=0$, $x_1=1/2$

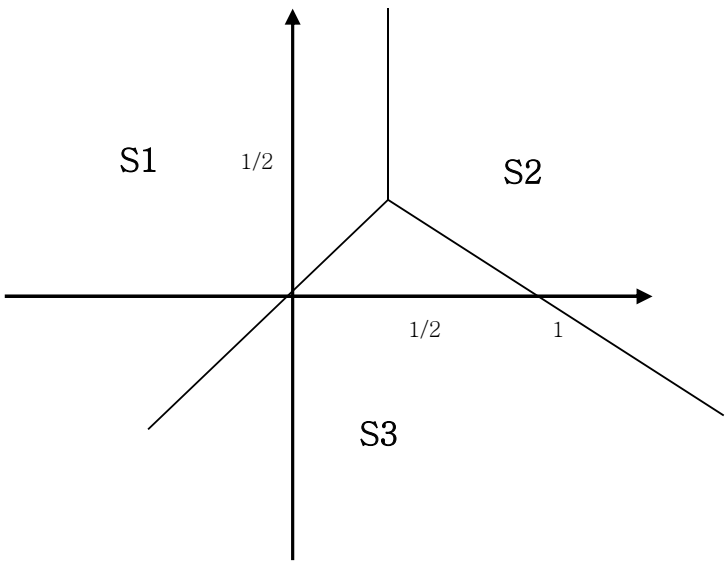
The decision boundary for S_2 over S_3

$(w_2^T- w_3^T)\underline{x}^{(a)}=0$, $[1\ 2\ -1][x_1\ x_2\ 1]^T=0$, $x_2=-1/2x_1+1/2$

The decision boundary for S_3 over S_1

$(w_3^T- w_1^T)\underline{x}^{(a)}=0$, $[1\ -2\ 0][x_1\ x_2\ 1]^T=0$, $x_2=1/2x_1$

The decision boundaries can be drawn as



According to the decision rule

$\underline{x} \in S_1$, if $g_1(\underline{x}) > g_2(\underline{x})$ and $g_1(\underline{x}) > g_3(\underline{x})$

$g_1(\underline{x}) - g_2(\underline{x}) > 0$ and $g_1(\underline{x}) - g_3(\underline{x}) > 0$

$-2x_1 + 1 > 0$ and $-x_1 + 2x_2 > 0$

$x_1 < 1/2$ and $x_2 > 1/2x_1$

Likewise for $\underline{x} \in S_2$ and $\underline{x} \in S_3$,

(b) Classify a point $\underline{x} = (1,1)^T$

Use the decision boundaries in the plot or $g_2(\underline{x}) > g_1(\underline{x})$ and $g_2(\underline{x}) > g_3(\underline{x})$

$\underline{x} \in S_2$

(c) A point that does not get classified

Let's try $\underline{x}=[1/2, 1/4]$

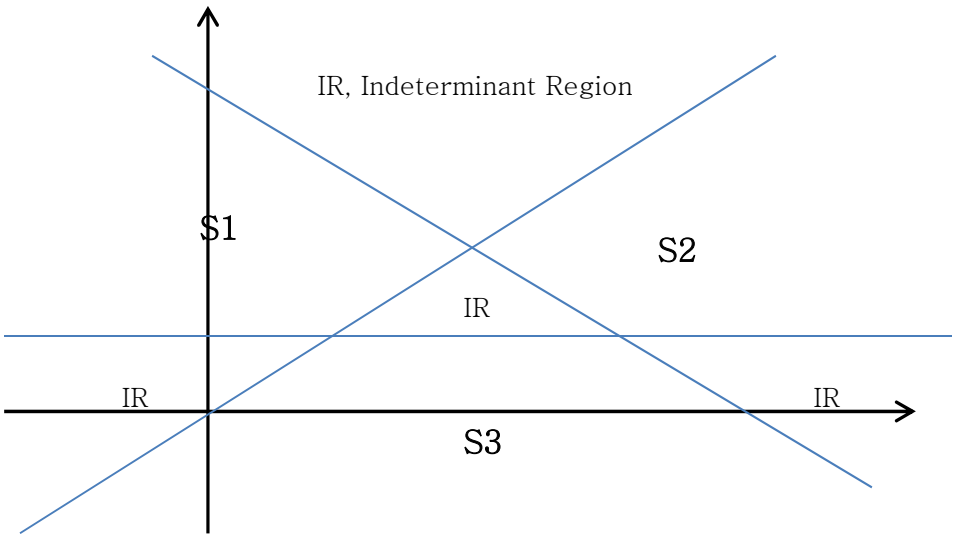
Get $g_1(\underline{x})=-1/4$, $g_2(\underline{x})=-1/4$, and $g_3(\underline{x})=-1/4$

Since $g_1(\underline{x})=g_2(\underline{x})=g_3(\underline{x})$

This point cannot get classified. Therefore this point becomes an indeterminant region. Moreover the points on each discriminant line cannot be classified because they make $g_k(\underline{x})=0$

2.

(a) Equating $g_i = 0$ give the decision boundaries **(b)** as given below.



(c) $\underline{x}^{(1)} = (6,5) \in S_2$, $\underline{x}^{(2)} = (2.5,2) \in \boldsymbol{R}$, and $\underline{x}^{(3)} = (3,0) \in S_3$.