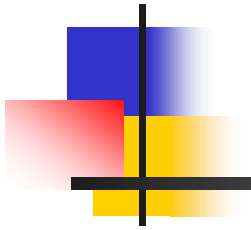


Similarity Measures

Distance and Similarity Measures

SIMILARITY AND DISSIMILARITY MEASURE FOR QUANTITATIVE FEATURES

Measures	Forms	Comments	Examples and Applications
Minkowski distance	$D_{ij}=\left(\sum_{l=1}^{d}\lvert x_{il}-x_{jl}\rvert^{1/n}\right)^{n}$	Metric. Invariant to any translation and rotation only for $n=2$ (Euclidean distance). Features with large values and variances tend to dominate over other features.	Fuzzy c-means with measures based on Minkowski family [130].
Euclidean distance	$D_{ij}=\left(\sum_{l=1}^{d}\lvert x_{il}-x_{jl}\rvert^{1/2}\right)^{2}$	The most commonly used metric. Special case of Minkowski metric at $n=2$. Tend to form hyperspherical clusters.	K-means algorithm [191]
City-block distance	$D_{ij}=\sum_{l=1}^{d}\lvert x_{il}-x_{jl}\rvert$	Special case of Minkowski metric at $n=1$. Tend to form hyperrecutangular clusters.	Fuzzy ART [57]
Sup distance	$D_{ij}=\max_{1\le l\le d}\lvert x_{il}-x_{jl}\rvert$	Special case of Minkowski metric at $n\to\infty$.	Fuzzy c-means with sup norm [39].
Mahalanobis distance	$D_{ij}=(\mathbf{x}_i-\mathbf{x}_j)^T\mathbf{S}^{-1}(\mathbf{x}_i-\mathbf{x}_j)$, where $\mathbf{S}$ is the within-group covariance matrix.	Invariant to any nonsingular linear transformation. $\mathbf{S}$ is calculated based on all objects. Tend to form hyperellipsoidal clusters. When features are not correlated, squared Mahalanobis distance is equivalent to squared Euclidean distance. May cause some computational burden.	Ellipsoidal ART [13], Hyperellipsoidal clustering algorithm [194].
Pearson correlation	$D_{ij}=(1-r_{ij})/2$, where $r_{ij}=\frac{\sum_{l=1}^{d}(x_{il}-\bar{x}_i)(x_{jl}-\bar{x}_j)}{\sqrt{\sum_{l=1}^{d}(x_{il}-\bar{x}_i)^2\sum_{l=1}^{d}(x_{jl}-\bar{x}_j)^2}}$	Not a metric. Derived from correlation coefficient. Unable to detect the magnitude of differences of two variables.	Widely used as the measure for analyzing gene expression data [80].
Point symmetry distance	$D_{ir}=\min_{\substack{j=1,\ldots,N\\ j\neq i}}\frac{\lVert(\mathbf{x}_i-\mathbf{x}_r)+(\mathbf{x}_j-\mathbf{x}_r)\rVert}{\lVert(\mathbf{x}_i-\mathbf{x}_r)\rVert+\lVert(\mathbf{x}_j-\mathbf{x}_r)\rVert}$	Not a metric. Compute the distance between an object $\mathbf{x}_i$ and a reference point $\mathbf{x}_r$. D_{ir} is minimized when a symmetric pattern exists.	SBKM (Symmetry-based K-means) [264].
Cosine similarity	$S_{ij}=\cos\alpha=\frac{\mathbf{x}_i^T\mathbf{x}_j}{\lVert\mathbf{x}_i\rVert\lVert\mathbf{x}_j\rVert}$	Independent of vector length. Invariant to rotation, but not to linear transformations.	The most commonly used measure in document clustering [261].

Similarity Measurements

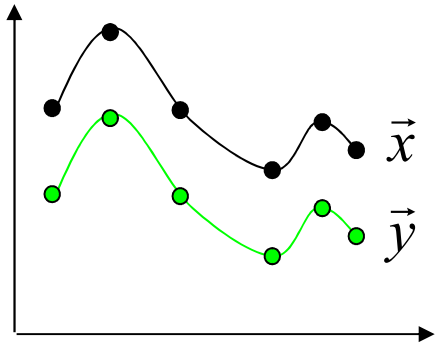
- Pearson Correlation

Two profiles (vectors) $\vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}$ *and* $\vec{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix}$

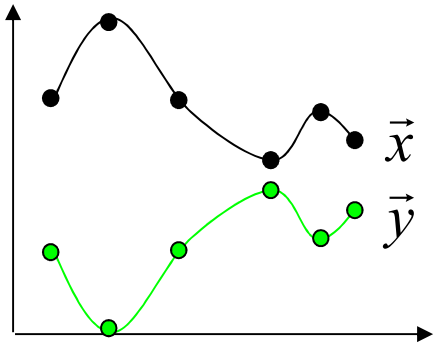
$$C_{pearson}(\vec{x},\vec{y}) = \frac{\sum_{i=1}^{N}(x_i - m_x)(y_i - m_y)}{\sqrt{[\sum_{i=1}^{N}(x_i - m_x)^2][\sum_{i=1}^{N}(y_i - m_y)^2]}}$$

$$m_x = \frac{1}{N}\sum_{n=1}^{N} x_n$$

$$m_y = \frac{1}{N}\sum_{n=1}^{N} y_n$$

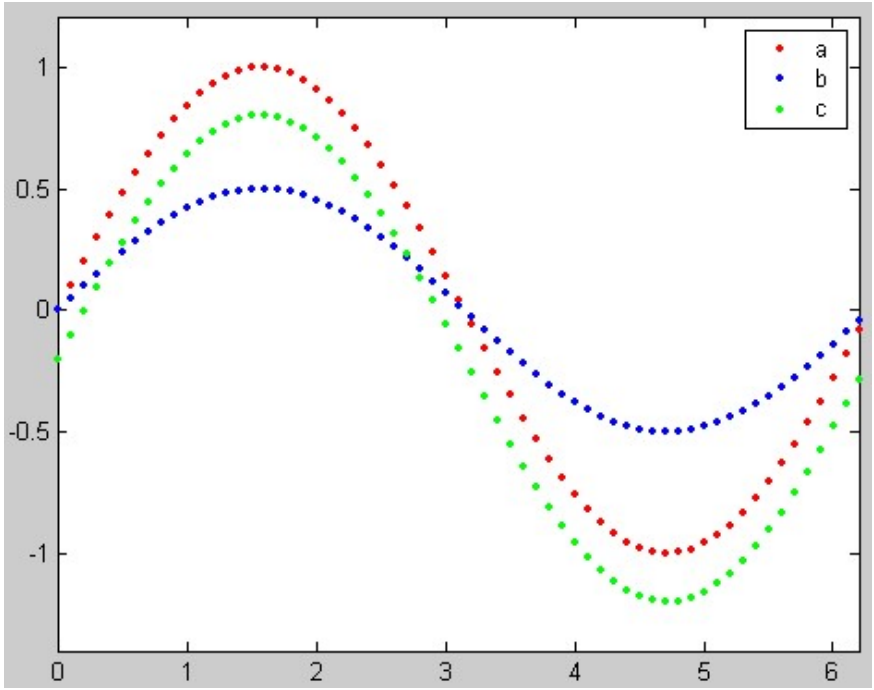


+1 ≥ Pearson Correlation ≥ – 1



Similarity Measurements

- Pearson Correlation: Trend Similarity



$$\vec{b} = 0.5\vec{a}$$

$$\vec{c} = \vec{a} - 0.2$$

$$C_{pearson}(\vec{a},\vec{b}) = 1$$

$$C_{pearson}(\vec{a},\vec{c}) = 1$$

$$C_{pearson}(\vec{b},\vec{c}) = 1$$

Similarity Measurements

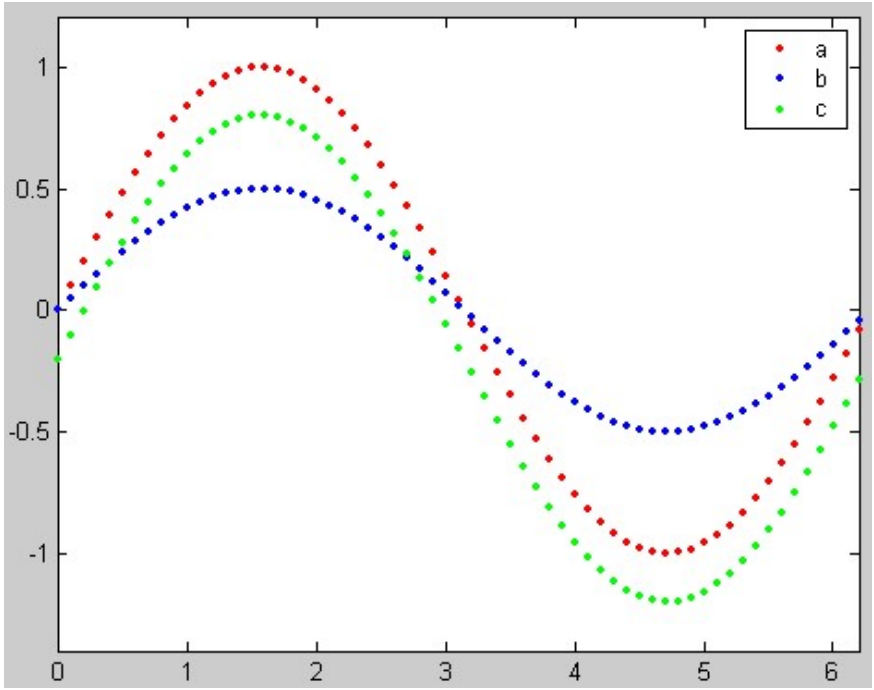
- Euclidean Distance

$$\vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix} \quad \vec{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix}$$

$$d(\vec{x}, \vec{y}) = \sqrt{\sum_{n=1}^{N} (x_n - y_n)^2}$$

Similarity Measurements

- Euclidean Distance: Absolute difference



$$\vec{b} = 0.5\vec{a}$$

$$\vec{c} = \vec{a} - 0.2$$

$$d(\vec{a},\vec{b}) = 2.8025$$

$$d(\vec{a},\vec{c}) = 1.5875$$

$$d(\vec{b},\vec{c}) = 3.2211$$

Similarity Measurements

- Cosine Correlation

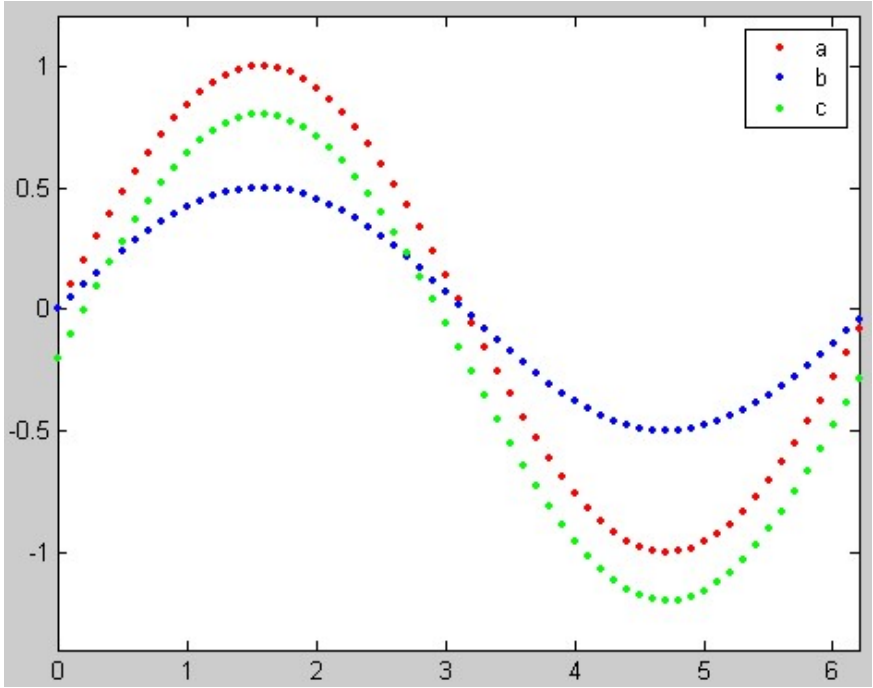
$$\vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix} \qquad \vec{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix}$$

$$C_{\text{cosine}}(\vec{x}, \vec{y}) = \frac{\frac{1}{N}\sum_{i=1}^{N} x_i \times y_i}{\|\vec{x}\| \times \|\vec{y}\|}$$

$\vec{x} = \vec{y}$ *+1 ≥ Cosine Correlation ≥ – 1* $\vec{x} = -\vec{y}$

Similarity Measurements

- Cosine Correlation: Trend + Mean Distance



$$\vec{b} = 0.5\vec{a}$$

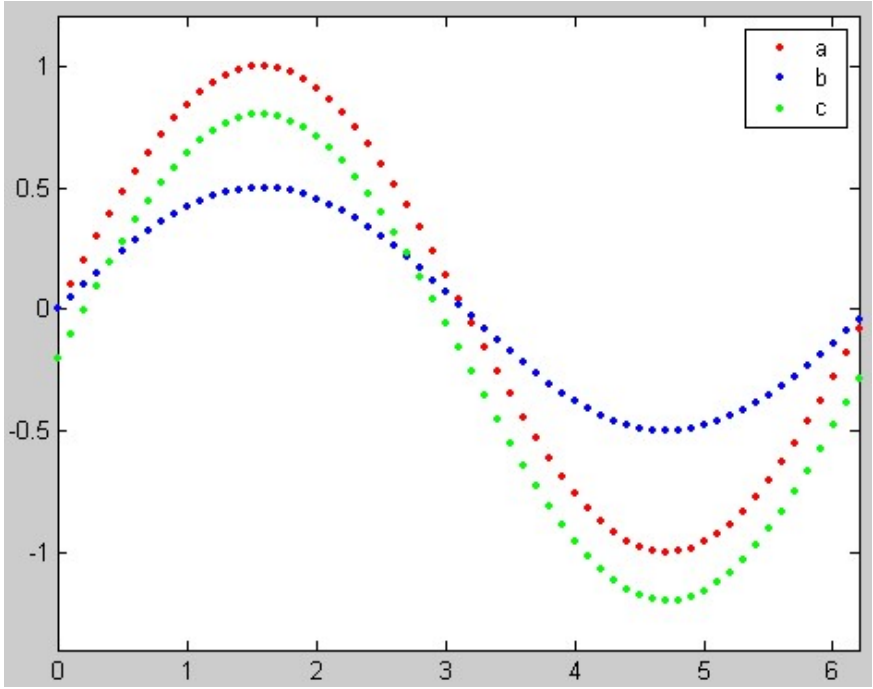
$$\vec{c} = \vec{a} - 0.2$$

$$C_{\cos ine}(\vec{a},\vec{b}) = 1$$

$$C_{\cos ine}(\vec{a},\vec{c}) = 0.9622$$

$$C_{\cos ine}(\vec{b},\vec{c}) = 0.9622$$

Similarity Measurements



$$\vec{b} = 0.5\vec{a}$$

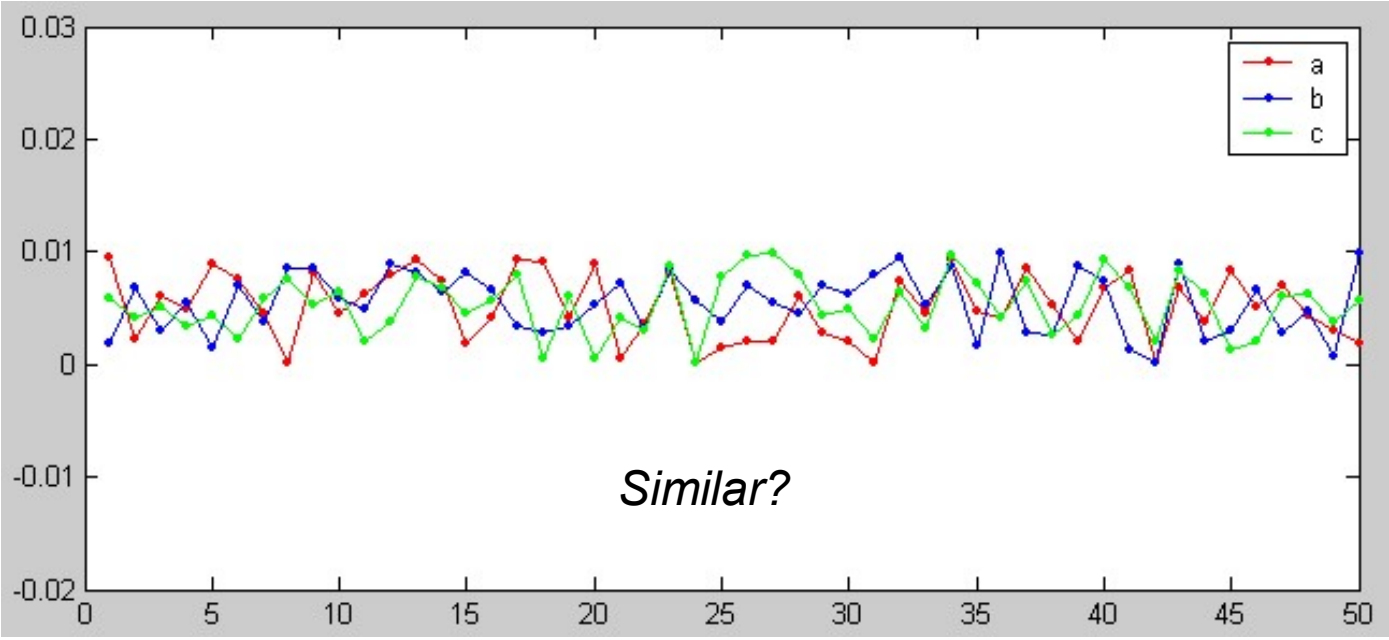
$$\vec{c} = \vec{a} - 0.2$$

$C_{pearson}(\vec{a},\vec{b}) = 1$ $\quad d(\vec{a},\vec{b}) = 2.8025$ $\quad C_{\cos ine}(\vec{a},\vec{b}) = 1$

$C_{pearson}(\vec{a},\vec{c}) = 1$ $\quad d(\vec{a},\vec{c}) = 1.5875$ $\quad C_{\cos ine}(\vec{a},\vec{c}) = 0.9622$

$C_{pearson}(\vec{b},\vec{c}) = 1$ $\quad d(\vec{b},\vec{c}) = 3.2211$ $\quad C_{\cos ine}(\vec{b},\vec{c}) = 0.9622$

Similarity Measurements



$C_{pearson}(\vec{a},\vec{b}) = -0.1175$ $d(\vec{a},\vec{b}) = 0.0279$ $C_{\cos ine}(\vec{a},\vec{b}) = 0.7544$

$C_{pearson}(\vec{a},\vec{c}) = 0.1244$ $d(\vec{a},\vec{c}) = 0.0255$ $C_{\cos ine}(\vec{a},\vec{c}) = 0.8092$

$C_{pearson}(\vec{b},\vec{c}) = 0.1779$ $d(\vec{b},\vec{c}) = 0.0236$ $C_{\cos ine}(\vec{b},\vec{c}) = 0.844$