原著論文

Information Diffusion and δ-Closeness Centrality

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Abstract

I consider the issue of optimal targeting in information diffusion networks. The initial information possessor is to target a single node so as to diffuse the information to all other nodes most effectively. For the purpose, the concept of closeness centrality may be useful, but if the value from delayed information is discounted by a discount factor, the concept should be properly modified. With this respect, I propose a modified concept of closeness centrality which I will call δ-(closeness)-centrality. The δ-centrality of a node is defined by the sum of discounted values generated from information transmission starting from the node given discount factor δ. Some advantages of δ-centrality over the closeness centrality are discussed.

Keywords: Closeness Centrality, Degree Centrality, δ-Closeness-Centrality, Information Diffusion

1. Introduction

People often obtain information from their friends, neighbors, colleagues, or other acquaintances. The information is transmitted from a person to others gradually via social links in the form of word-of-mouth communication. The spreads of rumors, religions and information about a product are typical examples. The information transmission may be proceeded either slowly or rapidly in a natural sense, but if there is someone who initially possesses the information and wants to spread it, his/her concern would be, in many cases, to diffuse it in the most effective way, and this could be achieved by targeting the right person as the initial information transmitter. Here, I use the phrase “the most effective information diffusion” in a rather ambiguous sense, since the meaning may vary, depending on the context. However, at the least, it does not necessarily mean ``the most rapid information diffusion." To motivate the main argument, consider the following example of consumer referral *à la* Jun *et al*. (2006a).[1)] A finite number of consumers form a connected network. Initially, none of them has information on the product of a seller. Thus, the seller does not obtain any value (its

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profit) from the relations with the uninformed consumers. If a consumer happens to be informed of the product, the information can be transmitted to his/her neighbors, and then the information is further spread from the neighbors to the neighbors' neighbors and so on. Now, if the seller is beginning to sell its product to the consumers, whom should it target to diffuse the information to the rest of the consumers most effectively?[2)] What is the seller's optimal targeting strategy? What is important in this example is that the value (profit) to the seller is not realized at once, but sequentially whenever a new consumer gets informed and buys. Moreover, the value from delayed information is discounted (by the seller) in proportion to the delay time according to some discount factor.[3)] Therefore, the aforementioned issue is not a straightforward problem when the realized values are of sequential nature and discounted accordingly. With regards to the issue of the most rapid information diffusion in a network, the concept of closeness centrality may serve as a useful criterion.[4)] In the social network theory, a node is said to have the highest closeness centrality if the sum of the shortest distances from it to all other nodes - which is its farness - is the minimum of the farness of all nodes. This notion of centrality is regarded as a measure of “how long it will take for information to spread from a given node to the other nodes in the network”.[5)] The underlying rationale for the desirability of information spreading is that nodes obtain some value (gain) from the information. For example, news about the danger of milk products containing the industrial chemical melanin has the social value in the sense that the information could help the society members take precautionary measures to avoid the danger. For fast information diffusion to be desirable, however, the value generated from the information should be more discounted as the transmission time gets longer; otherwise, all nodes in a connected network would be equally desirable as an initial target, since all of them could be reached after all and thus the value from the information is fully realized.[6)] If the values are discounted, the most central node should be varied, depending on the discount factor $\boldsymbol{\delta}$. Based on this argument, I define a modified concept of closeness centrality, which I call $\boldsymbol{\delta}$-(closeness)-centrality. The $\boldsymbol{\delta}$-centrality of a node is defined by the sum of the discounted value generated from information transmission starting from the node given discount factor $\boldsymbol{\delta}$. As $\boldsymbol{\delta}$ approaches zero, the $\boldsymbol{\delta}$-closeness-centrality becomes identical to the degree centrality which is defined by the degree of a node.[7)] As $\boldsymbol{\delta}$ approaches one, all nodes are made equally central. The argument so far suggests that a seller employing network marketing should target the node of the highest $\boldsymbol{\delta}$-closeness-centrality rather than a hub, which has the highest degree centrality. Although I make the main argument from the viewpoint of a private seller (who cares only about its own profit) in this paper, the same logic can be applied to the public central planner who is concerned about the welfare of the community members. As long as information is a socially valuable public good so that the welfare of the community is increased as more community members share it,[8)] it will be in the interest of the central planner to diffuse the information to more members more rapidly, just as it is in the interest of the private seller.[9)] The paper is organized as follows. Section 2 introduces some basic concepts in graph theory. In Section 3, I describe an illustrative model of targeted marketing. In Section 4, I define δ-closeness-centrality and its family. Section 5 contains the conclusion.

2. Terminology in Graph Theory

Let N be a (finite) set of nodes. A network (on N), denoted by g, is defined by a finite set of pairs of nodes in N called links.[10] That is, $\boldsymbol{g} = \{\boldsymbol{ij} | \boldsymbol{i}, \boldsymbol{j} \ni \boldsymbol{N}, \boldsymbol{i} \neq \boldsymbol{j}\}$. The interpretation of each link, $\boldsymbol{ij} \in \boldsymbol{g}$, is that i and j directly communicate with each other. If $\boldsymbol{ij} \in \boldsymbol{g}$, nodes i and j are said to be adjacent or neighbors. Let N_i be the set of i's neighbors. Then, the degree of a node $\boldsymbol{i} \in \boldsymbol{N}$, is defined by the number of its neighbors, i.e., $|\boldsymbol{N_i}|$, and denoted by *deg(i)*. It is said that there is a path from i to j if either $\boldsymbol{ij} \in \boldsymbol{g}$, or there exist distinct nodes $i_1, i_2, \cdots, i_m (\neq i, j)$ such that $ii_1, i_1 i_2, \cdots i_m j \in g$. A network is called connected if there is a path from i to j for all i and $j \in N$ and otherwise, it is called not connected or disconnected. The distance between i and j is defined by the number of links in the shortest path (or geodesic) between them and denoted by $t(i, j)$. If there is no path between i and j, I define the distance by $t(i, j) = \infty$. Also, $\max_{j \in N} t(i, j)$ is called the eccentricity of i. A cycle is defined by a path such that $i = j$.[11] A connected network which does not contain a cycle is called a tree. The path between any pair of nodes in a tree is unique.

3. Illustrative Model

A seller (or a firm) is thinking of selling its products to a set of consumers N forming a connected network g. Consumer i obtains the utility v_i (from the product) which is an independently and identically distributed random variable with the distribution function $F(v_i)$ for all $i \in N$, but none of them are initially informed of the availability of the product. Once Consumer i is informed, he/she buys the product if and only if $v_i \geq p$ where p is the price. Thus, the seller's value of transmitting the information to the buyer is $(p-c)[1-F(p)]$ where c is the unit production cost. Here, $1-F(p)$ is the probability that Consumer i buys the product at price p. I normalize $(p-c)[1-F(p)]$ to be one.[12] Now, I can consider the following multi-period interaction between the seller and the consumers. In the first period, the seller targets a single consumer who will be informed of the product by the contact with the seller. The information is transmitted to the other consumers by word-of-mouth communication. That is, once the initial consumer gets informed, the information is passed on to his/her neighbors in the next period, and then it is successively passed on to the neighbors' neighbors in the next period, until all the consumers in the network are informed. The seller is not perfectly patient so as to care about rapid information transmission, meaning that it discounts the future profit. Discounting is usually formulated in two ways; by a fixed ratio and by a fixed amount. For fixed ratio discounting, a discount factor $\delta(>0)$ is used, while a discount cost $\kappa(>0)$ is used for fixed amount discounting. Note that this information diffusion process ensures the shortest path information transmission. Since information is diffused in all possible directions, no node misses the information via the shortest path before it receives the information via a longer path. To illustrate, consider the network structure drawn in Figure 1. It contains cycles, implying that there can be multiple paths between some pair of consumers. For example, there are three paths from Consumer 4 to Consumer 7. If Consumer 4 is picked as the initial information disseminator, the information is transmitted from Consumer 4 to Consumer 7 in two periods which corresponds to the geodesic distance between them. The information reached in three periods via Consumer 8 is meaningless to Consumer 7 since it is redundant. The geodesic path may not be unique in a general network, although it is unique in any tree.

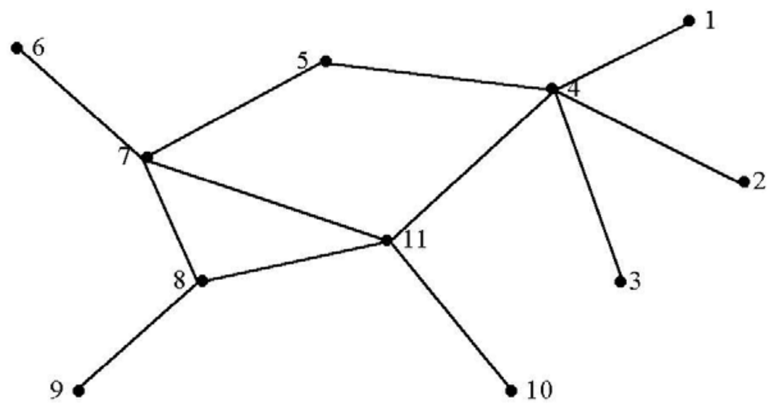

Figure 1

4. δ-Closeness-Centrality and Its Family

For expositional purpose, I use the framework of consumer referral, but the arguments below will be valid for general information diffusion processes. To follow the notation which was defined in Section 2, let N be the set of nodes (consumers) and g be a network defined on N. In this section, I do not restrict my attention to connected networks, but consider all possible networks for the sake of generality.

4.1 Fixed Ratio Discounting

Let Node i be targeted. Under fixed ratio discounting, the value to the seller, which comes up with the sum of the expected discounted profits, is

$$C(i;\delta) = \sum_{j \in N \setminus \{i\}} \delta^{t(i,j)}$$

$$= \sum_{t=1}^{\infty} \delta^t n_i(t), \quad (1)$$

where $n_i(t) = |N_i(t)|$ and $N_i(t) = \{j \in N \mid t(i,j) = t\}$.[13] This measure does not simply consider the number of neighbors to Node i, but considers all nodes with a higher weight on nodes closer to Node i. The weights $\{\delta^t\}$ geometrically decrease with respect to the distance from Node i. I will call $C(i;\delta)$ the δ-(closeness)-centrality of Node i. This is a measure of the value generated from a targeted node given discount factor δ. That is, it measures how valuable Node i is as the information disseminator when discounting the value of delayed information. Then, I say that Node i has the higher δ-centrality than node j if $C(i;\delta) > C(j;\delta)$, and that it is δ

-central, if it has the highest δ-centrality, i.e., $i = \arg\max_{j\in N} C(j;\delta)$. In order to compare this concept with the usual closeness centrality, recall the following definition of the latter:

$$C(i) = \left[\sum_{j\in N\setminus\{i\}} t(i,j) \right]^{-1} = \left[\sum_{t=1}^{\infty} t n_i(t) \right]^{-1}. \tag{2}$$

Suppose $\delta = 1$. Then, it is clear that $C(i;1) = \sum_{t=1}^{\infty} n_i(t) = |N| - 1$ for all $i \in N$ for any network. This means that all nodes would be equally central for $\delta = 1$. On the other hand, suppose $\delta = 0$. Then, clearly $C(i;0) = 0$ for all $i \in N$ for any networks. Again, all nodes would be the same in terms of centrality. This suggests that it is reasonable to restrict the attention to $\delta \in (0,1)$. Below, I will provide a limiting result establishing the relations of δ-centrality with the two widely used centrality concepts; closeness centrality and the degree centrality.

Proposition 1 *(i) There exists* $\underline{\delta}(>0)$ *such that for all* $\delta \le \underline{\delta}$, $C(i;\delta) > C(j;\delta)$ *whenver* $deg(i) > deg(j)$. *(ii) There exists* $\overline{\delta}(<1)$ *such that for all* $\delta \ge \overline{\delta}$, $C(i;\delta) > C(j;\delta)$ *whenver* $C(i) > C(j)$.

Proof. See the appendix.

Proposition 1 says that if a node has the highest degree centrality, it is δ-central for δ sufficiently close to zero, and that if a node has the highest closeness centrality, it is δ-central for δ sufficiently close to one. In other words, this proposition says that δ-centrality is implied by degree centrality if $\delta \to 0$ and implied by closeness centrality if $\delta \to 1$. One obvious advantage of δ-closeness-centrality over the usual closeness-centrality is that the criterion can be applied to any class of networks, while the closeness-centrality criterion has no selection power among disconnected networks. To see this, resort to the definition of closeness-centrality given in equation (2). If g is not connected, there must be a pair of nodes (j,k) such that $t(j,k) = \infty$, implying that $C(i) = 0$ for any $i \in N$, for any disconnected g. This is deemed as a large drawback of closeness-centrality. However, δ-centrality usually differs across nodes even in the disconnected network, because computing $C(i;\delta)$ needs the values of each $t(j,k)$, not the value of $\sum_{j\in N\setminus\{i\}} t(i,j)$. If the centrality concept depends on $\sum_{j\in N\setminus\{i\}} t(i,j)$ as closeness centrality, the centrality would be the same for all nodes, even if $t(i,j) = \infty$ for some pair (i,j). However, it would not be a problem in $C(i;\delta)$. For instance, in a disconnected network illustrated in Figure 2, δ-centrality of Node 1 is highest for all δ, while all nodes have the same closeness centrality. Table 1 demonstrates this point. Now, it is worthwhile to compare the two concepts, $C(i)$ and $C(i;\delta)$, more closely. Note that a high value of $C(i)$ does not necessarily imply a high value of $C(i;\delta)$. For example, if $C(i) > C(j)$ but Node j has more direct neighbors than Node i, it is possible that $C(j;\delta) > C(i;\delta)$. This suggests that it is important to consider the distribution of

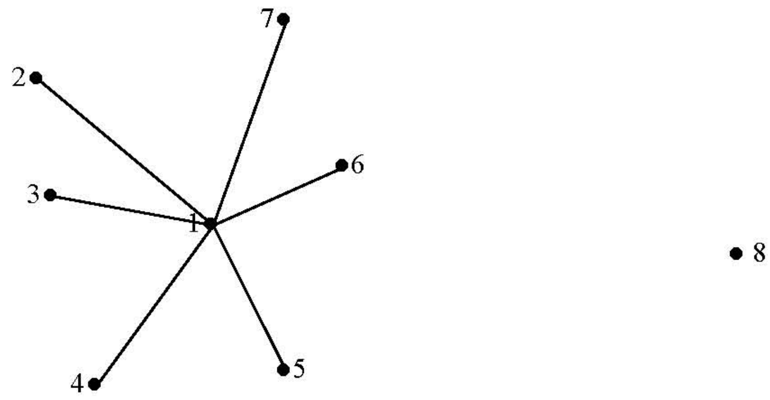

Figure 2. Disconnected Network

		$C(i,\delta)$	$C(i)$
i	1	6δ	0
	2	$\delta+5\delta^2$	0
	3	$\delta+5\delta^2$	0
	4	$\delta+5\delta^2$	0
	5	$\delta+5\delta^2$	0
	6	$\delta+5\delta^2$	0
	7	$\delta+5\delta^2$	0
	8	0	0

Table 1: δ-centrality for a disconnected network

distances from a node rather than the simple sum of the distances in identifying the highest δ-central node. By taking this into consideration, I give two sufficient conditions for a node to be δ-central. For the purpose, I will say that the distribution $\{n_i(t)\}_{t=1}^{\infty}$ first-order stochastically dominates (FOSD) the distribution $\{n_j(t)\}_{t=1}^{\infty}$, if and only if $\sum_{\tilde{t}=1}^{t} n_i(\tilde{t}) \geq \sum_{\tilde{t}=1}^{t} n_j(\tilde{t})$ for all t, and denote the relation by $n_i(t) \succ n_j(t)$.[14] Also, I will say that the distribution $\{n_i(t)\}_{t=1}^{\infty}$ is a mean-preserving spread of the distribution $\{n_j(t)\}_{t=1}^{\infty}$, if and only if for some $m, \Delta \in \{1,2,\cdots\}$,

$$n_i(t) = \begin{cases} n_j(t) + m & \text{if } t = t_0 - \Delta \\ n_j(t_0) - 2m & \text{if } t = t_0 \\ n_j(t) + m & \text{if } t = t_0 + \Delta \\ n_j(t) & \text{otherwise.} \end{cases}$$

Intuitively speaking, a mean-preserving spread of a distribution makes the distribution more dispersed while keeping the mean equal to the original distribution.[15)]

Proposition 2 *For* $\delta \in (0,1)$ *, Node* i *has the higher* δ *-centrality than Node* j *if* $n_i(t) \succ n_j(t)$*. Node* i *is* δ*-central if* $n_i(t) \succ n_k(t)$ *for all* $k \in N \setminus \{i\}$.

Proof. See the appendix.

This proposition provides a useful guideline for comparing δ-centrality of two nodes with different closeness-centrality.[16)] However, since it is just a sufficient condition for δ-centrality, not the necessary and sufficient condition, it only helps to eliminate some first-order stochastically dominated nodes as a candidate for the δ-central node. The δ-central node may not satisfy the first-order stochastic dominance over all other nodes. For example, in Figure 1, $x_{11}(t) \succ x_7(t)$, and as a result, $C(11;\delta) > C(7;\delta)$ for all δ. This simply implies that Node 7 cannot be δ-central for any δ. Figure 3 shows a transitional change in the δ-central node with respect to a change in δ. Node 4 is δ-central for $\delta < 0.5$, while node 11 is for $\delta > 0.5$.[17)]

Proposition 3 *For* $\delta \in (0,1)$*, Node* i *has the higher* δ*-centrality than Node* j *if* $n_i(t)$ *is a mean-preserving spread of* $n_j(t)$.

Proof. See the appendix.

In a contrast with Proposition 2, Proposition 3 is useful for comparing δ-centrality of two nodes with the same closeness-centrality. For example, in Figure 1, although Node 5 and Node 8 have the same closeness-centrality, Node 8 has a higher δ-centrality than Node 5, since the distribution of the former is a mean-preserving spread of that of the latter. For this, see Table 2. The intuition behind this proposition is that δ^t is a convex function of t. Two things are noteworthy. First, it is not computationally too demanding to identify the δ-central node. In fact, just as identifying the highest central node, identifying the highest δ-central node only requires the information about $n_i(t)$ for all i and t which can be readily obtained from the geodesic distance matrix.[18)] Second, δ-centrality has the stronger selection power than the degree centrality or the closeness centrality. For instance, Node 7 and Node 11 have the same degree centrality, but Node 11 has the higher δ-centrality for all $\delta > 0$. Also, as shown before, Node 8 has the higher δ-centrality than Node 5 for $\delta = 0.6$, while they have the same closeness centrality. It might deserve discussing information

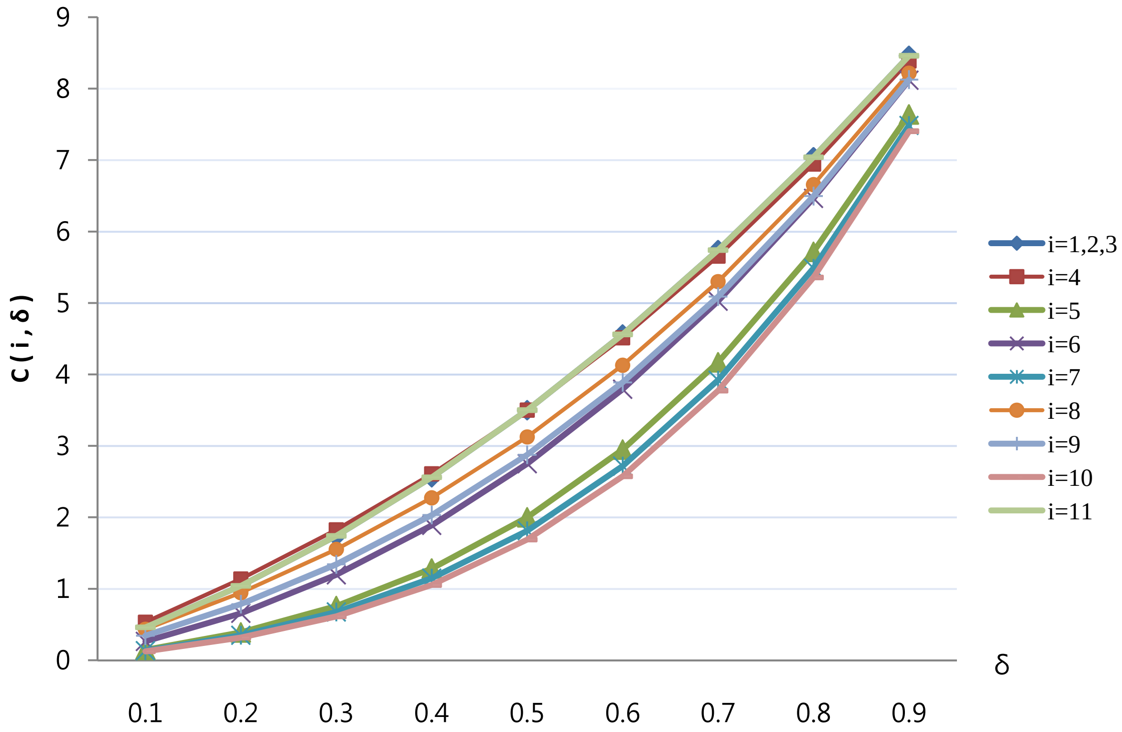

Figure 3 δ-Central Node

		t					
		1	2	3	4	$C(i,\delta)$	$C(i)$
i	1	1	4	3	2	2.9472	1/26
	2	1	4	3	2	2.9472	1/26
	3	1	4	3	2	2.9472	1/26
	4	5	3	2	0	4.512	1/17
	5	2	6	2	0	3.792	1/20
	6	1	3	3	3	2.7168	1/28
	7	4	3	3	0	4.128	1/19
	8	3	4	3	0	3.888	1/20
	9	1	2	4	3	2.5728	1/29
	10	1	3	6	0	2.976	1/25
	11	4	6	0	0	4.561	1/16

Table 2: δ-centrality for Figure 1 ($\delta = .6$)

centrality as a related concept. The concept, which was developed by Stephenson and Zelen (1989), considers the possibility of information transmission via all possible paths between i and j not only via geodesics between them. Note that δ-centrality considers the information transmission only through the geodesic.[19)] The definition of information centrality implicitly assumes that all paths do not have the same information. In the context of consumer referral, however, only the information via geodesics is meaningful, although the information flows via all possible paths insofar as the information transmitted via all paths has the same content, because only the fastest arriving information matters. This is a crucial difference between the two concepts.

4.2 Fixed Amount Discounting

Under fixed amount discounting, a delay in information transmission reduces the value (generated from the information) to the seller by κt. Thus, the seller's total expected profit in this case when it chooses Node i as the target is

$$\tilde{C}(i;\kappa) = |N| - 1 - \kappa \sum_{j \neq i} t(i,j). \tag{3}$$

If $\kappa = 0$, the seller is indifferent among all nodes as under fixed ratio discounting for any connected network, since all nodes are reachable eventually without any loss. Otherwise, however, maximizing $\tilde{C}(i;\kappa)$ is equivalent to minimizing $\sum_{j \neq i} t(i,j)$ for any $\kappa \neq 0$, implying that the node of the highest closeness centrality $C(i)$ is the favorite to the seller. This argument suggests that the closeness centrality can be justified under nonzero fixed amount discounting. Note that this concept still has the selection power only among connected networks.

5 Conclusion

In this paper, I introduced δ-centrality which is a variation of closeness centrality developed in the social network theory, and argued that this concept is relevant in the individualistic model in which values are generated sequentially whenever the information reaches a node and they are discounted by a fixed discount factor. The concept of δ-closeness centrality can be usefully applied to the problem of finding the optimal targeting strategy when the person initially possessing information knows the structure of the network, but cannot be used when he/she does not know the structure. There is a well-known strategy, so-called acquaintance immunization strategy, though, to find a hub even in the case of the unknown network structure.[20] Just pick a random neighbor (acquaintance) of a random node. Then, it is most likely to be a hub, since the hub has the largest number of neighbors, implying that the first randomly selected node is most likely to be a neighbor of a hub. However, to the best of my knowledge, virtually no method is known about how to find the node of the highest closeness centrality. I believe that this will be a promising future research project.

Appendix

Proof of Proposition 1:

(i) If $\delta \to 0$, I have $\lim_{\delta \to 0} C(i;\delta)/C(j;\delta) = n_i(1)/n_j(1) = deg(i)/deg(j)$ for all $i, j \in N$.

This means that

$$deg(i) > deg(j) \Rightarrow \lim_{\delta \to 0} C(i;\delta) > \lim_{\delta \to 0} C(j;\delta),$$

implying that $C(i;\delta) > C(j;\delta)$ if $deg(i) > deg(j)$ for δ in a neighborhood of zero.

(ii) By using a Taylor expansion of definition (1) around $\delta = 1$, I have

$$\lim_{\delta\to 1}\frac{C(i;\delta)}{C(j;\delta)} = \lim_{\delta\to 1}\frac{\sum_{k\in N\setminus\{i\}}(1+t(i,k)(\delta-1))}{\sum_{k\in N\setminus\{j\}}(1+t(j,k)(\delta-1))}$$

$$= \lim_{\delta\to 1}\frac{N-1+C(i)^{-1}(\delta-1)}{N-1+C(j)^{-1}(\delta-1)}$$

$$= \frac{C(i)}{C(j)}.$$

Therefore, $C(i;\delta) > C(j;\delta)$ if $C(i) > C(j)$ for δ in a neighborhood of one.

Proof of Proposition 2:

Let $x_i(t) \equiv n_i(t)/(|N|-1)$ be the ratio of nodes with $t(i,j) = t$ in $N\setminus\{i\}$. Then, $\{x_i(t); t = 1,2,\cdots\}$ can be interpreted as a probability distribution in the sense that $x_i(t) \ge 0$ for all $t \ge 1$ and $\sum_{t=1}^{\infty} x_i(t) = 1$. According to the conventional definition, the probability distribution $x_i(t)$ first-order stochastically dominates (FOSD) the probability distribution $x_j(t)$, if and only if $\sum_{\tilde{t}=1}^{t} x_i(\tilde{t}) \ge \sum_{\tilde{t}=1}^{t} x_j(\tilde{t})$ for all t. Note that I am using FOSD of $n_i(t)$ and that of $x_i(t)$ interchangeably. Let $F(t)$ and $G(t)$ be two distributions of t.

Lemma 1 $F(t) \succ G(t)$ implies $E[\delta^t; F(t)] > E[\delta^t; G(t)]$ for all $\delta \in (0,1)$.

Proof of Lemma 1: Let $H(t) = F(t) - G(t)$. Integrating by parts, I have

$$\int_0^{\infty}\delta^t dH(t) = \left[\delta^t H(t)\right]_0^{\infty} - \ln\delta\int_0^{\infty}\delta^t H(t)dt.$$

Since $H(0) = 0$ and $H(t) = 0$ for large t, it follows that $\left[\delta^t H(t)\right]_0^{\infty} = 0$, which in turn implies that $\int_0^{\infty}\delta^t dH(t) \ge 0$ iff $\ln\delta\int_0^{\infty}\delta^t H(t)dt \le 0$, i.e., $\int_0^{\infty}\delta^t H(t)dt \ge 0$. Since $H(t) \ge 0$ for all t, the result follows.

The proof of Proposition 2 is immediate from the definition of δ-centrality and Lemma 1.

Proof of Proposition 3:

A distribution $F(t)$ is said to "second-order stochastically dominate (SOSD)" a distribution $G(t)$ if for every nonincreasing convex function u, the following holds;

$$\int_0^{\infty}u(t)dF(t) \ge \int_0^{\infty}u(t)dG(t).$$

If $x_i(t)$ is a mean-preserving spread of $x_j(t)$, the former can be obtained from the latter by randomizing $x_j(t)$ in a manner that the new realization is $t+z$ where z has a distribution $H(z)$ with a mean of zero, i.e., $\int_0^\infty zdH(z)=0$. Then, for a nonincreasing convex $u(\cdot)$, I have

$$\int_0^\infty u(t)dx_j(t) = \int\left(\int_0^\infty u(t+z)dH(z)dx_i(t)\right)dx_i(t)$$

$$\leq \int_0^\infty u\left(\int_0^\infty (t+z)dH(z)\right)dx_i(t)$$

$$= \int_0^\infty u(t)dx_i(t),$$

since $\int_0^\infty zdH(z)=0$. This means that $x_i(t)$ SOSD $x_j(t)$. Since δ^t is a nonincreasing convex function of t, the proof is completed.

Notes

1) Also, see Jun & Kim (2008) and Jun *et al.* (2006b).
2) This is the question raised in the conclusion of Jun and Kim (2008).
3) The discount factor is defined by the present value of one dollar in the next period. Economists perceive it as an individual's personal characteristic, more specifically, his/her preference of the present over the future (reflecting the uncertainty about the future). So, discount factors for individual consumers or firms have been estimated by many economists. See, for example, Cameron and Gerdes (2003) for the estimation results. They used a strategy for the measurement of individual-specific discount factors via survey methods. Their main idea was to observe the choices of consumers among goods yielding a different profile of benefits and costs over time.
4) The closeness-based concept of centrality was first suggested by Bavelas (1948), and later generalized and more formalized by Hakami (1965), Sabidussi (1966) and Freeman (1979).
5) See, for example, Borgatti (2005) or Wasserman and Faust (1994).
6) By a more desirable target, I mean that it generates the higher value (profit) through the information diffusion.
7) The idea of using degree as an index of centrality was first proposed by Shaw (1954) and formally developed by Nieminen (1974) and Freeman (1979).
8) The news about milk products containing melanin will be a good example of elucidating this feature. However, some information could be socially harmful, albeit very rare. In this case, the harmful information is called a public *bad*.
9) In rare instances, the initial information disseminator may prefer slow information diffusion. This could occur when each node has the potential to be a competing seller once it receives the information or when the hidden undesirable feature of the information could be revealed in the long run. Rogers (1962) notes the possibility.
10) By the restriction of distinct pairs (and distinct nodes *resp.*) we disallow multiple edges (and loops *resp.*).
11) Since I mean only a simple network by a network, a loop which is a link from a node to itself is

disallowed.

12) The price p may be interpreted as the profit-maximizing one, that is, $p^{*} \equiv \arg\max_{p} (p-c)[1-F(p)]$.

13) The possibility that $t=\infty$ is a direct consequence of allowing disconnected networks.

14) Since $\{n_i(t)\}_{t=1}^{\infty}$ is not a probability distribution, I am abusing the terminology of first-order stochastic dominance here. For the exact definition of FOSD, see the proof of Proposition 1.

15) t is well known that a mean-preserving spread is equivalent to the second-order stochastic dominance (SOSD) which will be formally defined in the appendix. The proof is also provided in the appendix.

16) It is trivial to observe that $C(i) > C(j)$ if $\{n_i(t)\}$ FOSD $\{n_j(t)\}$.

17) To see this, note that $C(4;\delta) - C(11;\delta) = \delta(1-2\delta)(1-\delta) > 0$ iff $\delta < 0.5$.

18) There exist fast algorithms for computing the geodesic distance matrix from the adjacency matrix. For instance, UCINET is widely used software for the purpose.

19) *This feature is captured in the definition of $N_i(t)$ in the formula (1).

20) This strategy originates from Cohen, Havlin and Ben-Avrahan (2003) in physics.

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