

Ph.D. Dissertation

Capacity and Error Exponents for Multiple-Input
Multiple-Output Wireless Channels

(多重 送受信 안테나 無線채널의 情報理論的 特性에 관한 研究)

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School of Electrical Engineering and Computer Science
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in Partial Fulfillment of the Requirements for*

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by

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Abstract

Multiple-input multiple-output (MIMO) communication systems using multiple transmit and receive antennas have emerged as the vital breakthrough that offers significant promise in achieving high spectral efficiency and link reliability for future wireless communications. However, the true benefits of multiple antennas may be limited by two types of channel impairments: spatial fading correlation and rank deficiency of the channel. While spatial fading correlation reduces the diversity gain offered by multiple antennas, rank deficiency due to double scattering or the key-hole effect decreases the spatial multiplexing gain of multiple-antenna channels.

In this dissertation, taking into account realistic propagation environments in the presence of spatial fading correlation and rank deficiency, we study two important information-theoretic measures—channel capacity and error exponents—for MIMO wireless channels, which provide an enhanced understanding of the potential and limitations of those channels. The channel capacity is the crucial asset determining the fundamental limit on the achievable transmission rates that can be supported in a communication channel and giving an insight into the ability of transmission over the channel. The error exponent indicates how difficult it may be to achieve a certain level of reliability in communication at a transmission rate below the channel capacity or how complex the codes are required to achieve a certain level of reliable communications. By studying these two relevant information-theoretic aspects, we can obtain more complete insights into the ultimate limits to communications over realistic MIMO channels in terms of information transmission capability and reliability of communications.

The dissertation contains three main results. First, assuming that the channel is perfectly known at the receiver but unknown at the transmitter, we analyze MIMO capacity in Shannon sense, often termed the ergodic (or mean) capacity, for MIMO channels in the presence of spatial fading correlation, double scattering, and the

keyhole effect. Second, we derive exact analytical expressions for capacity statistics such as the probability density function (pdf), cumulative distribution function (cdf), characteristic function, mean, variance, skewness, kurtosis, and other higher-order statistics. We also investigate the impacts of spatial correlation and rank deficiency on the capacity statistics. These statistical results allow us to analytically evaluate another relevant information-theoretic capacity measure, often termed the outage capacity, as well as the ergodic capacity. In addition, non-Gaussianity of the capacity distribution is quantified as the Kullback-Leibler distance between the capacity distribution and its Gaussian approximation. Using the Gram-Charlier expansion of pdf, this quantitative measure of non-Gaussianity can be explicitly expressed in terms of the skewness and kurtosis of the capacity distribution. Third, Gallager's random coding and expurgated error exponents are derived for MIMO channels. The effects of channel coherence time, spatial correlation, and rank deficiency are considered. Also, from the random coding bound, the required code lengths for a certain probability of error are calculated to aid in the assessment of the required coding complexity over MIMO channels with or without channel impairments.

Keywords: Channel capacity, cutoff rate, double scattering, error exponent, keyhole effect, multiple antennas, multiple-input multiple-output (MIMO) system, spatial fading correlation.

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